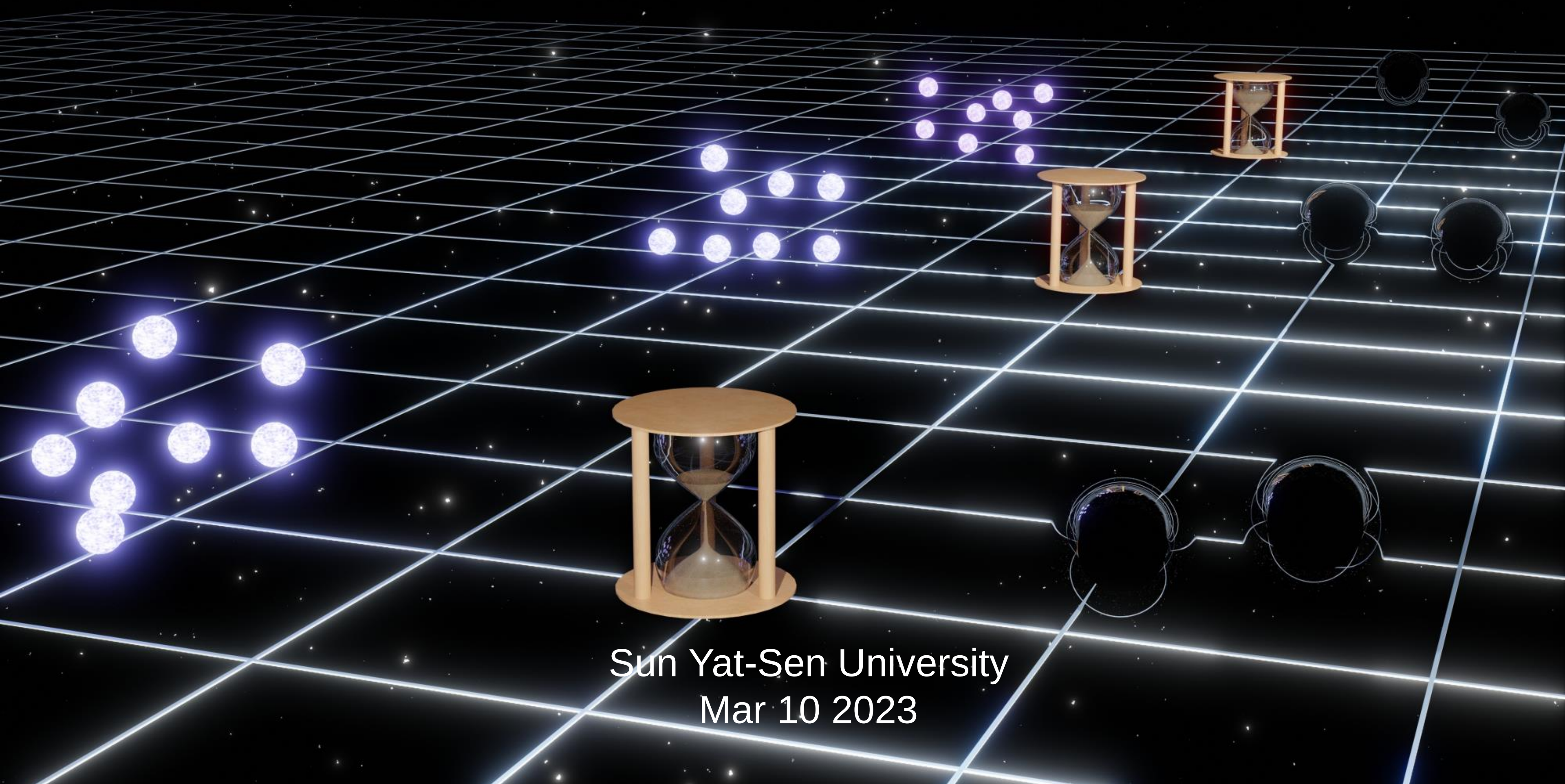
Measure the Universe with Cosmological Standard Timers

arXiv: 2112.10422 & 2206.03142

Qianhang Ding 丁乾航
The Hong Kong University of Science and Technology

With Yi-Fu Cai (USTC), Chao Chen (HKUST), Yi Wang (HKUST)



Sun Yat-Sen University
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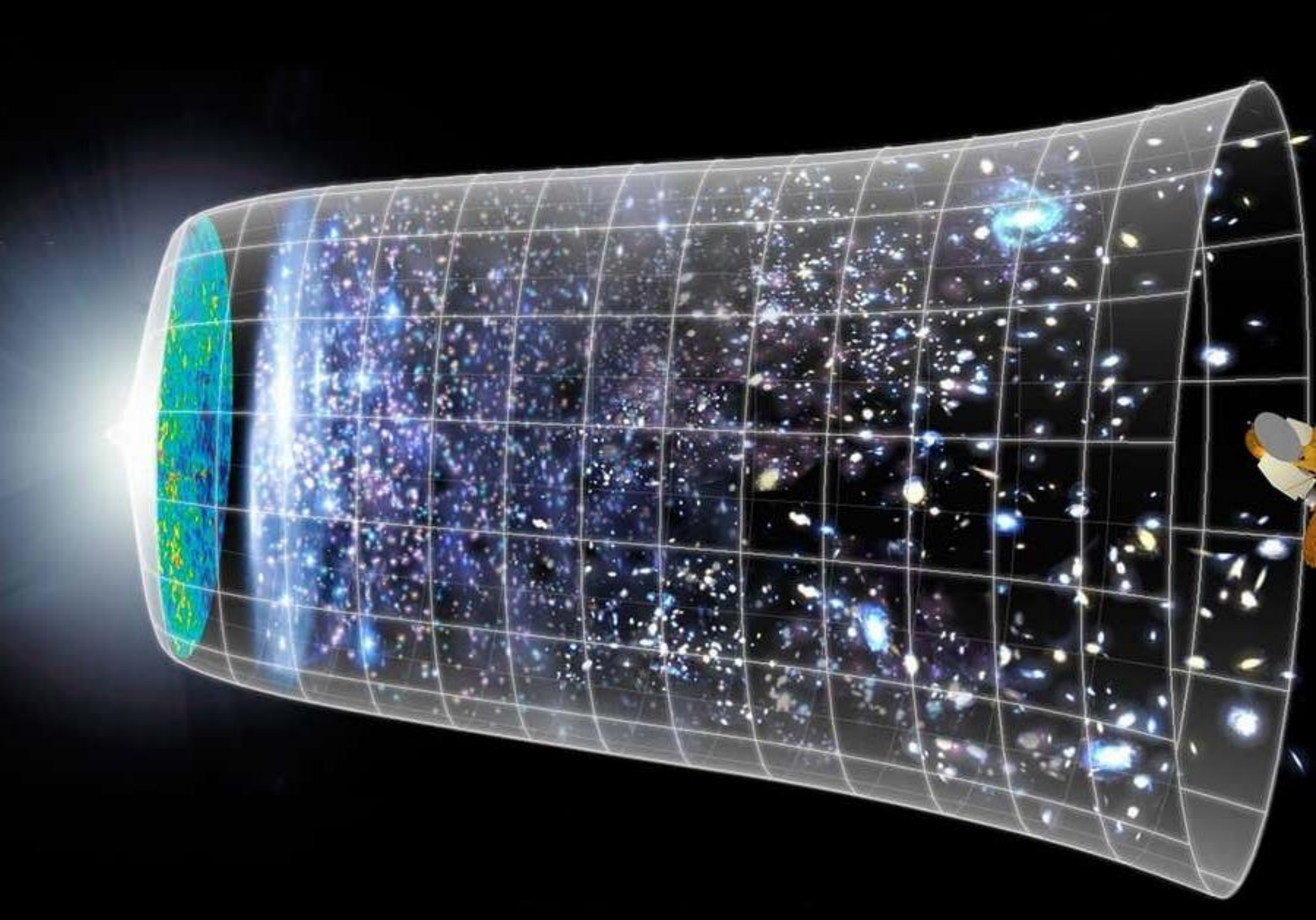


Image Credit: NASA

How to measure the Universe?

Image Credit: ESO

Standard Candle

$$F = \frac{L}{4\pi\, d_L^2(z)}$$

Standard Ruler

$$\theta = \frac{r_s}{D_M(z)}$$

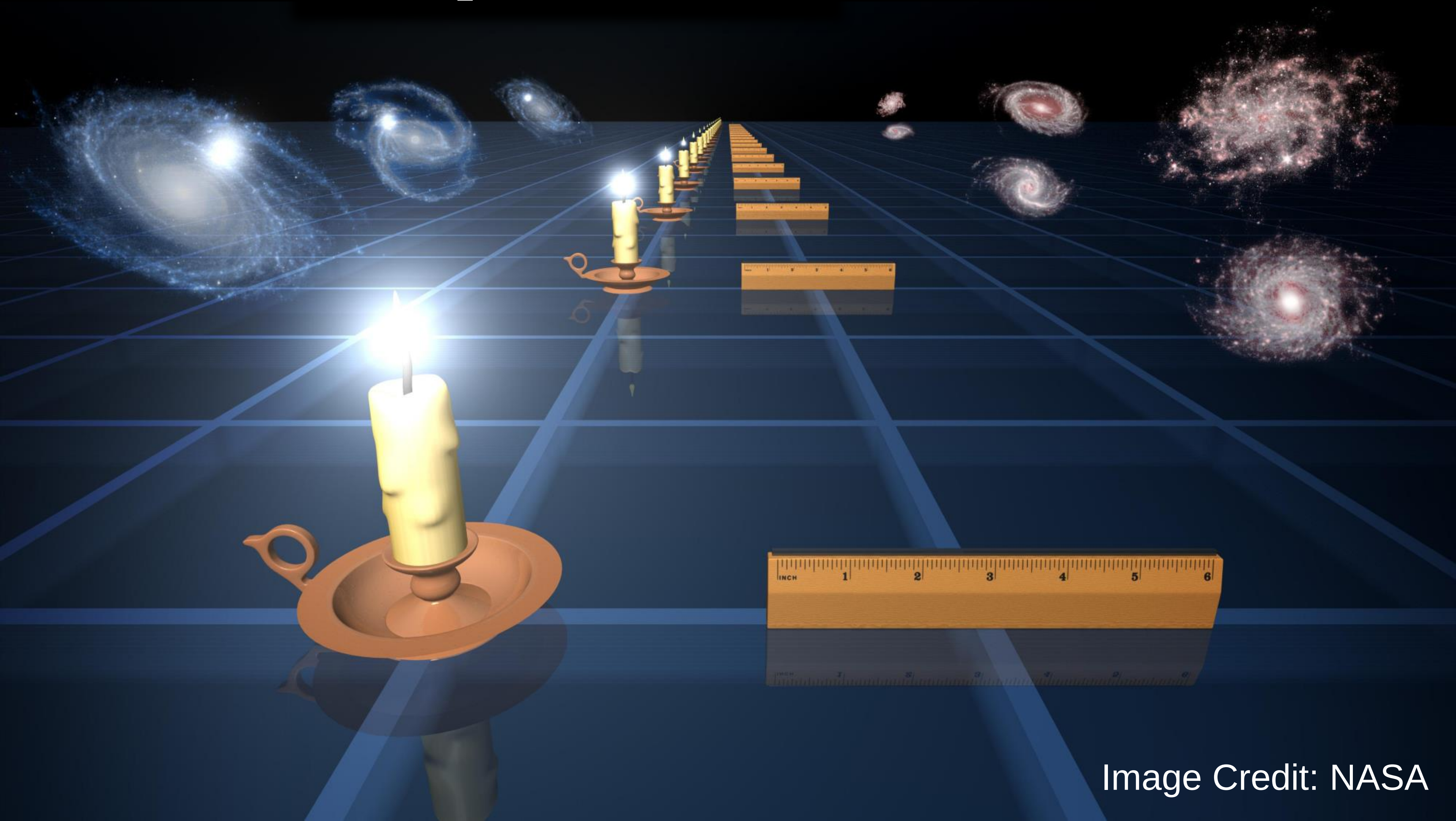
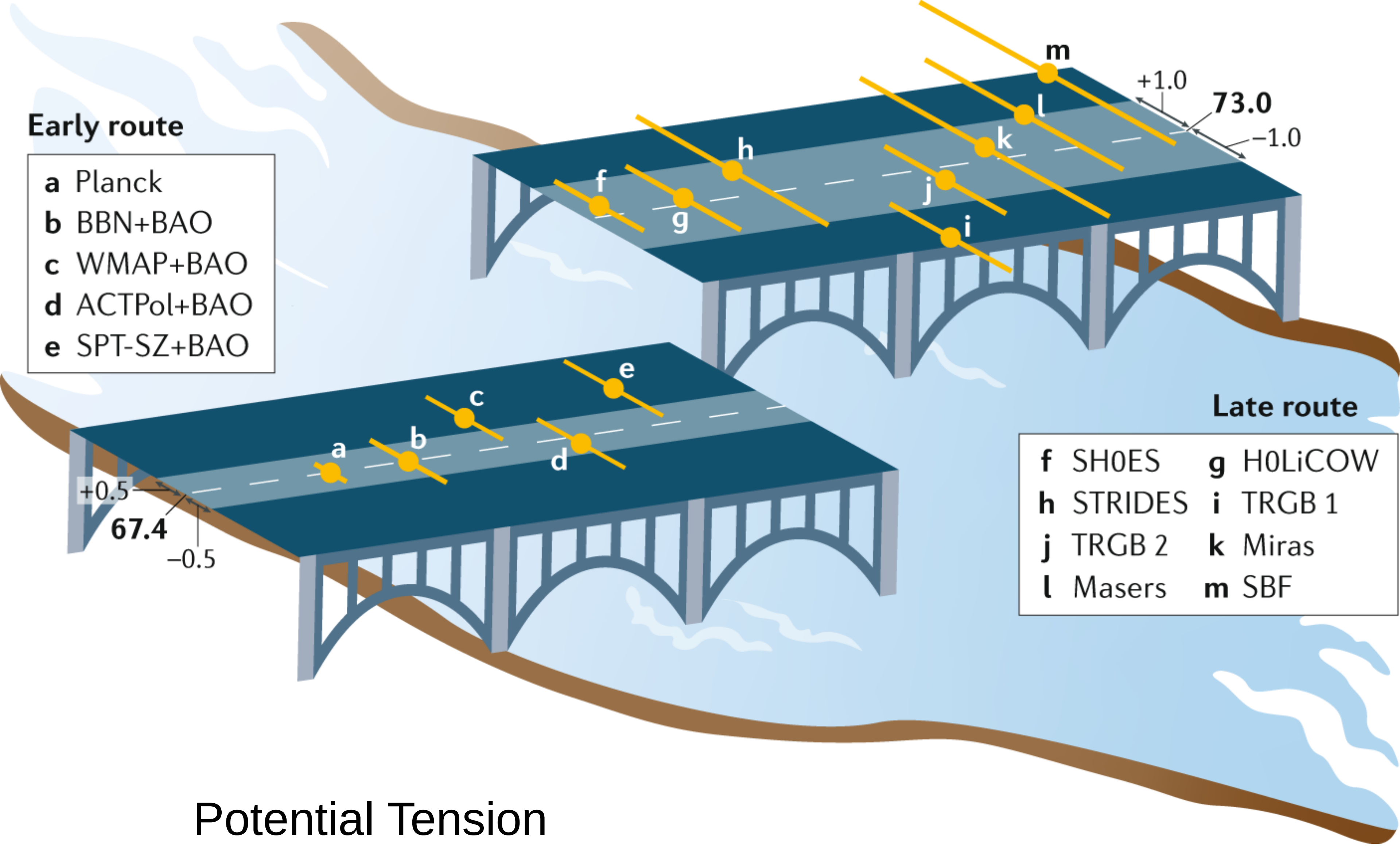


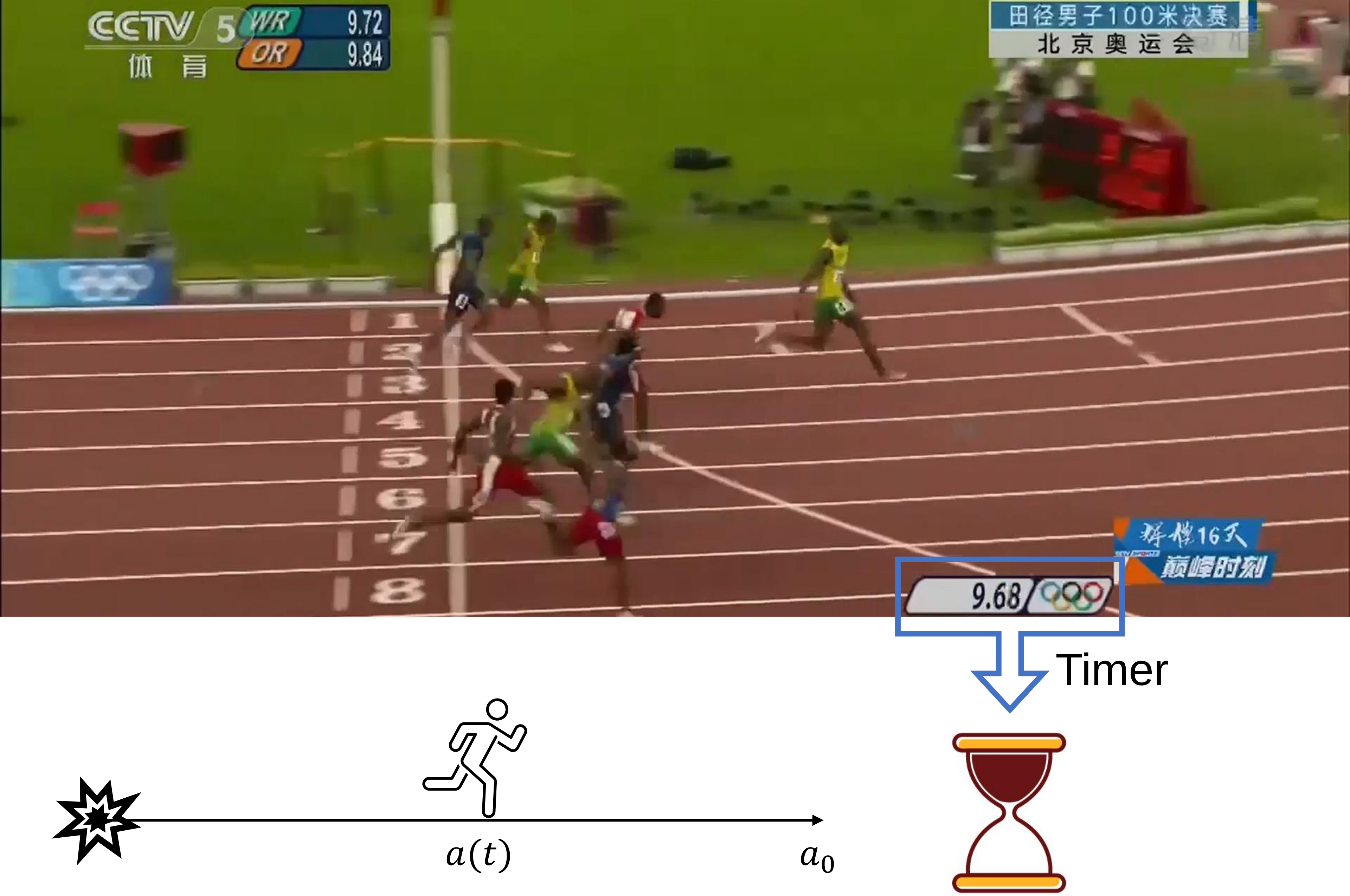
Image Credit: NASA



Potential Tension

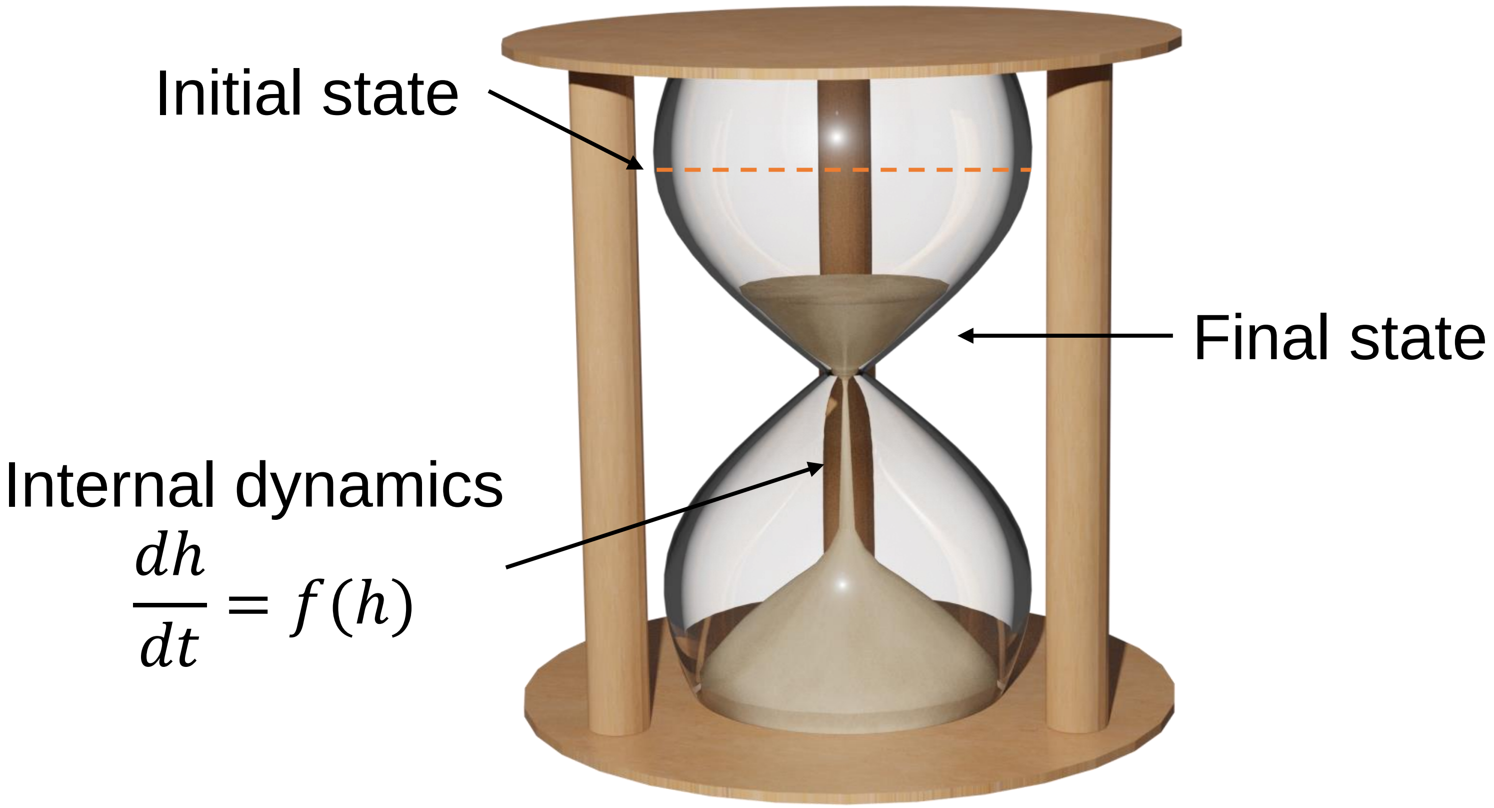
Riess, Adam G. "The expansion of the universe is faster than expected." *Nature Reviews Physics* 2.1 (2020): 10-12.

Another way to measure the Universe?



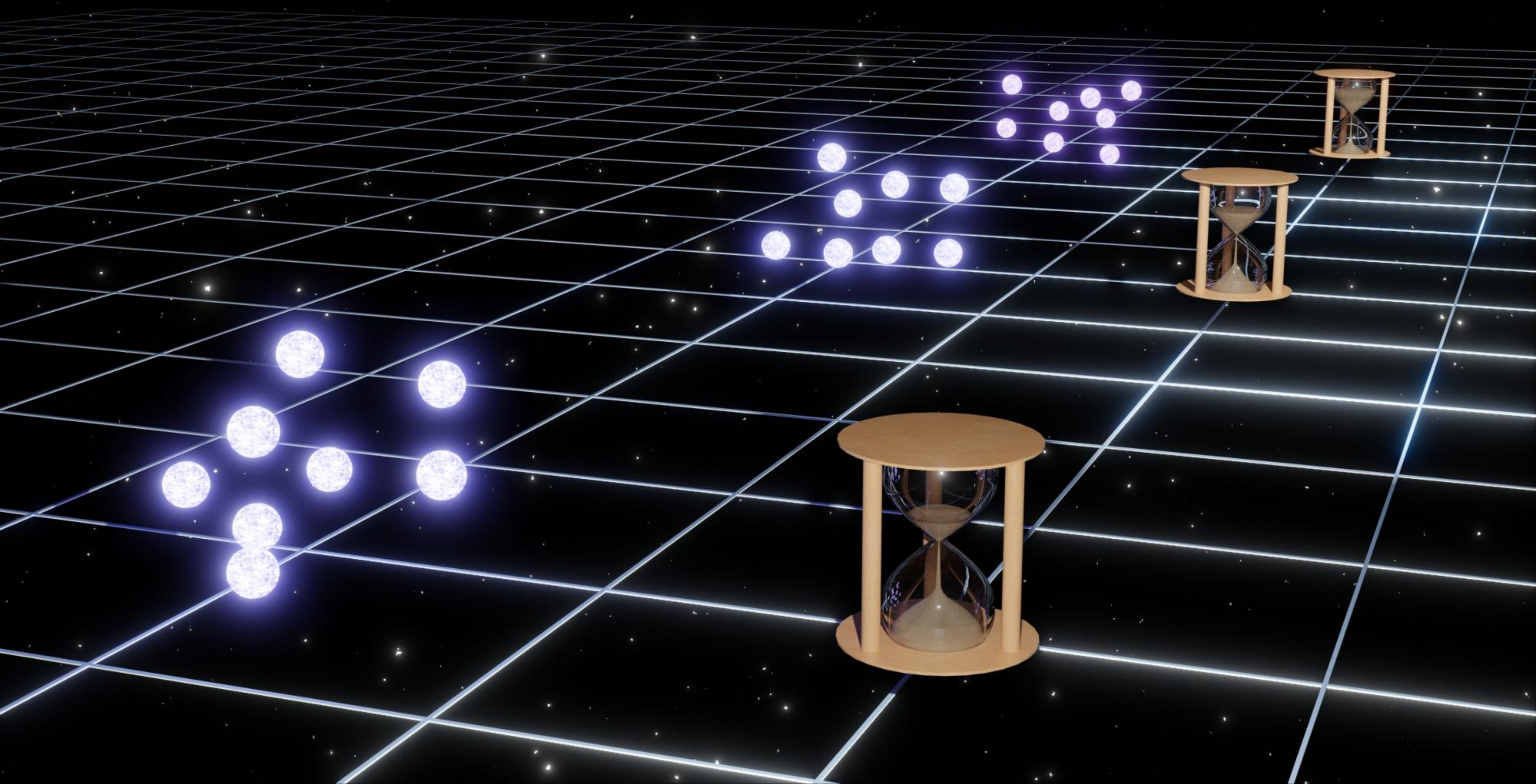
CCTV 5
体育
WR 9.72
OR 9.84
田径男子100米决赛
北京奥运会
9.68
Timer
a(t)
a0

How to know the elapsed time in the timer?

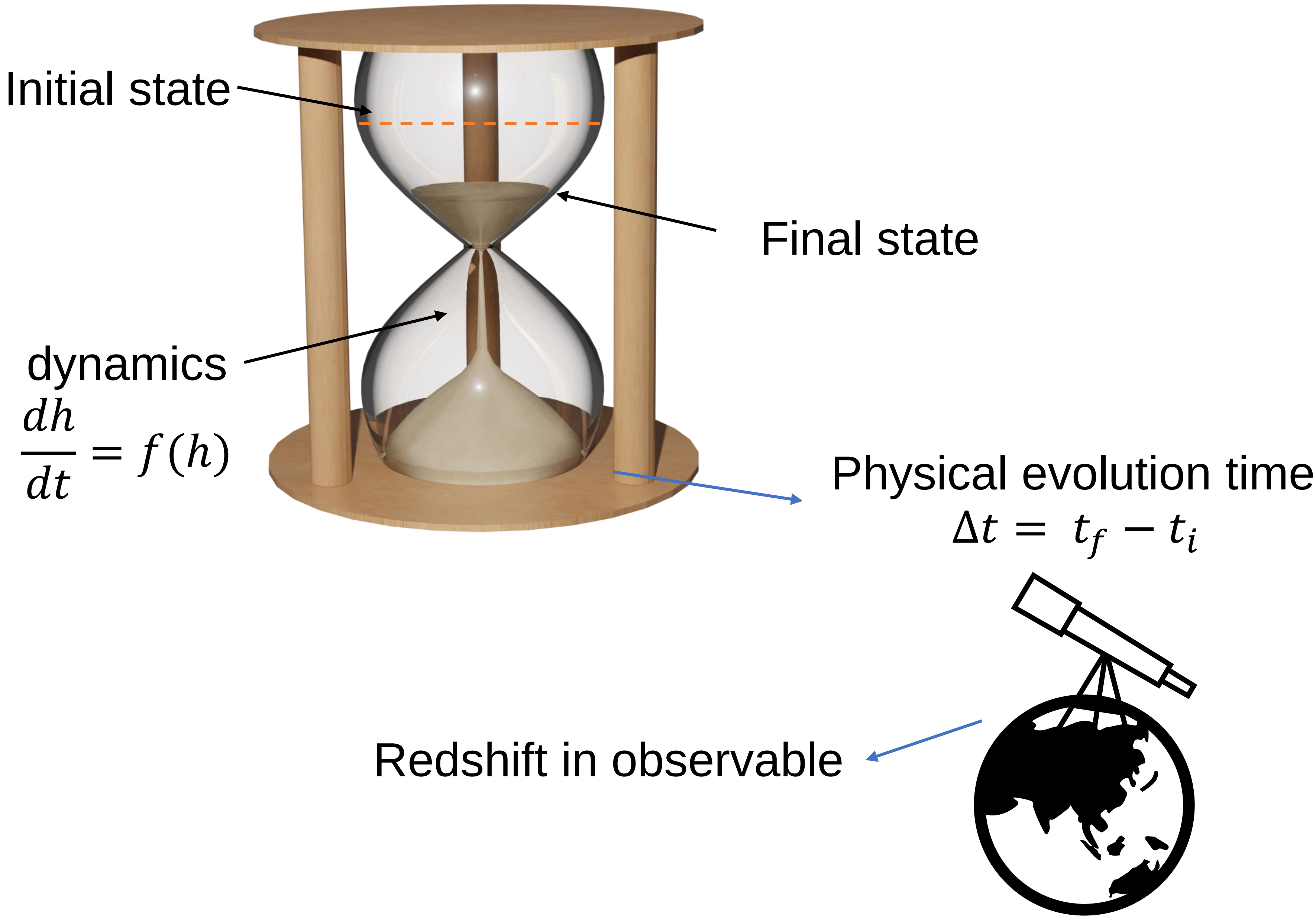

Initial state
Final state
Internal dynamics
$\frac{dh}{dt} = f(h)$

How to obtain $a(t)$?

$$1 + z(t) = \frac{a_0}{a(t)}$$



Standard timers in dynamical systems



A single parameter standard timer

Initial state: Initial statistical distribution of dynamical systems

$$S(M;t_{\rm i}) = \frac{dN}{dM_{\rm i}}$$

Dynamic: time evolution of parameter in dynamical systems

$$\frac{dM}{dt} = -f(M) \Rightarrow \int_{M_{\rm i}}^{M_t} \frac{dM}{f(M)} = g(M_t) - g(M_{\rm i}) = -\Delta t$$

Final state: Statistical distribution of dynamical systems at physical time t

$$S(M;t) = \frac{dN}{dM_{\rm i}}\frac{dM_{\rm i}}{dM_t} = S(M;t_{\rm i})\frac{g'(M_t)}{g'(g^{-1}(g(M_t)+\Delta t))}$$

Observed state: Redshifted statistical distribution of dynamical systems detected at redshift z

$$S_o(M_z;t) = S_o(M_z;t_{\rm i})\frac{g'(M_z)}{g'(g^{-1}(g(M_z)+\Delta t_z))}$$

Redshift-time relation: Comparing the observed state with the initial state gives the redshift-time relation

$$S_o(M_z;t) \simeq \begin{cases} S(M;t_{\rm i})\dfrac{dM_{\rm i}}{dM_{\rm i}(z)} & , \ g(M_z) \gg \Delta t_z \\ S_o(g^{-1}(\Delta t_z);t_{\rm i})\dfrac{g'(M_z)}{g'(g^{-1}(\Delta t_z))}, & g(M_z) \ll \Delta t_z \end{cases}$$

A multi-parameter standard timer

Initial state: Initial statistical distribution of dynamical systems

$$S(\mathbf{M}; t_{\rm i}) = \frac{dN}{d^n\mathbf{M}_{\rm i}}$$

Dynamic: time evolution of parameter in dynamical systems

$$\frac{d\mathbf{M}}{dt} = -\mathbf{f}(\mathbf{M})$$

Final state: Statistical distribution of dynamical systems at physical time t

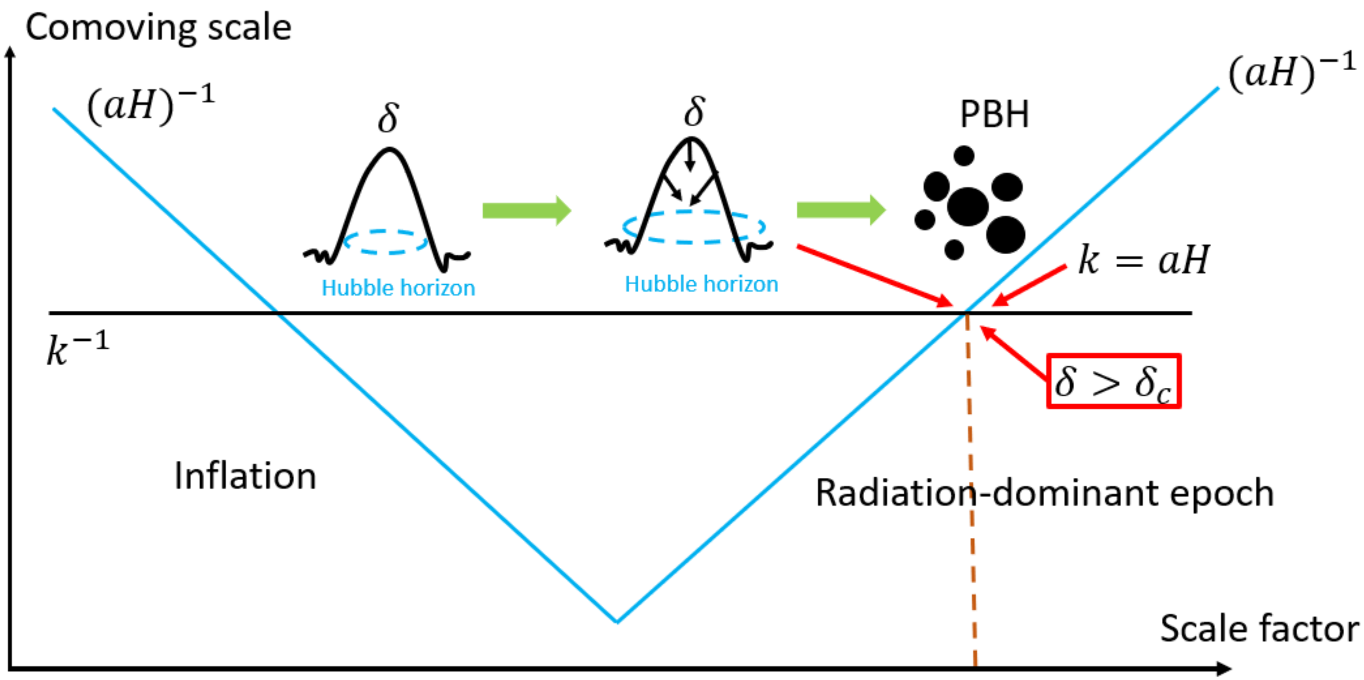
$$S(\mathbf{M}; t) = \frac{dN}{d^n\mathbf{M}_{\rm i}} \det \mathbf{J}(\mathbf{M}, \Delta t) = S(\mathbf{M}; t_{\rm i}) \det \mathbf{J}(\mathbf{M}, \Delta t)$$

$$\mathbf{J}_{ij} \equiv \partial M_i(t_{\rm i}) / \partial M_j(t)$$

Observed state: Redshifted statistical distribution of dynamical systems detected at redshift z

$$S_o(\mathbf{M}_z; t) = \frac{dN}{d^n\mathbf{M}_{\rm i}(z)} \det \mathbf{J}(\mathbf{M}_z, \Delta t_z)$$

Primordial black holes as a standard timer candidate



The Primordial origin gives an identical primordial mass function

$$n(M) = \frac{f_{\rm PBH}}{\sqrt{2\pi}\sigma M} \exp\left[-\frac{\ln^2(M/M_{pk})}{2\sigma^2}\right]$$

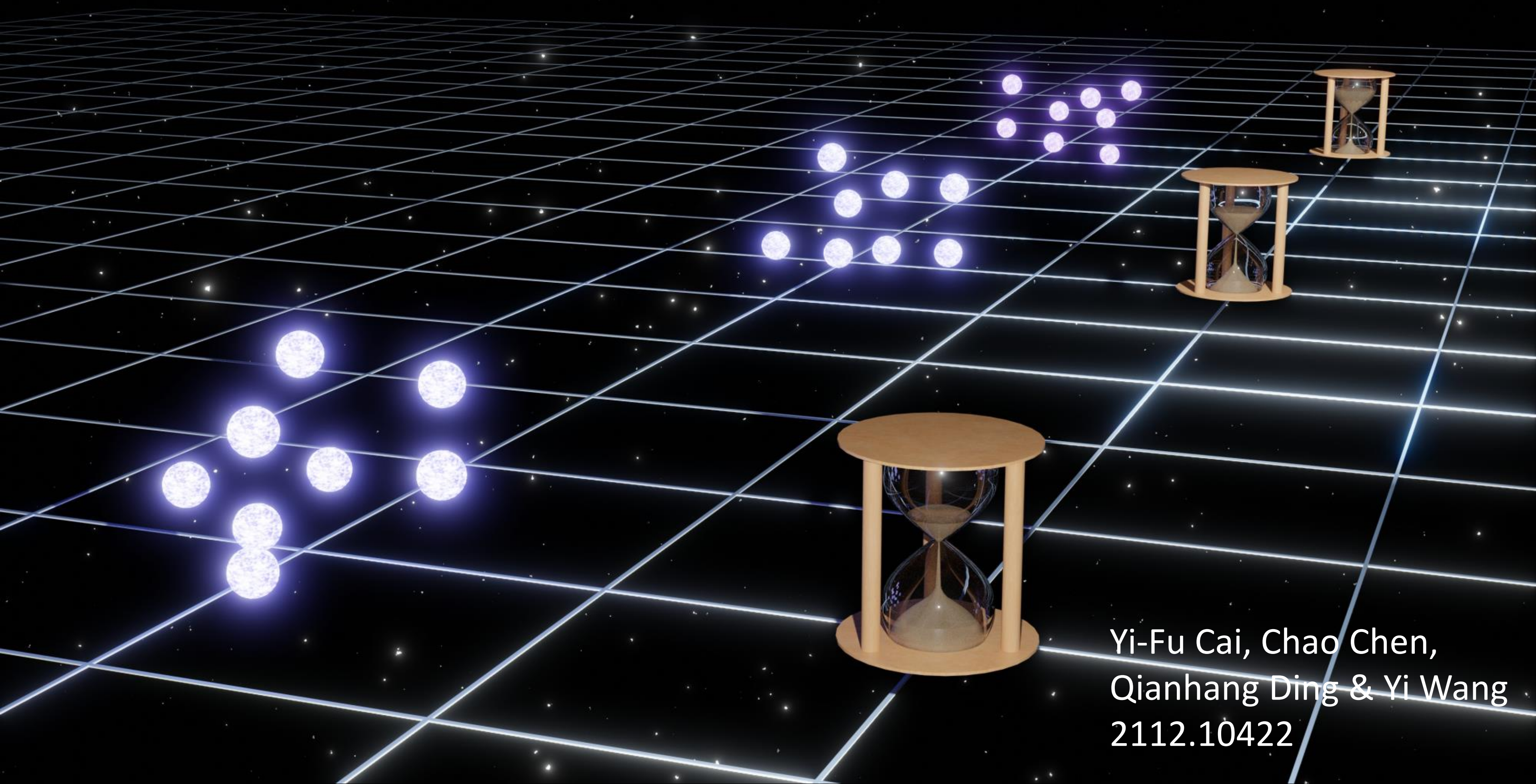
PBH binaries were formed with an identical probability distribution

$$\frac{dP}{da\,de} = \frac{3}{4} f_{\rm PBH}^{3/2} \frac{a^{1/2}}{\bar{x}^{3/2}} \frac{e}{(1-e^2)^{3/2}}$$

Standard Timers from Primordial Black Hole Clustering

The primordial mass function of PBHs

$$n(M) = \frac{f_{\mathrm{PBH}}}{\sqrt{2\pi}\sigma M} \exp\left[-\frac{\ln^2(M/M_{pk})}{2\sigma^2}\right]$$



Yi-Fu Cai, Chao Chen,
Qianhang Ding & Yi Wang
2112.10422

How to extract the physical evolution time?

The evolution of the PBH mass function

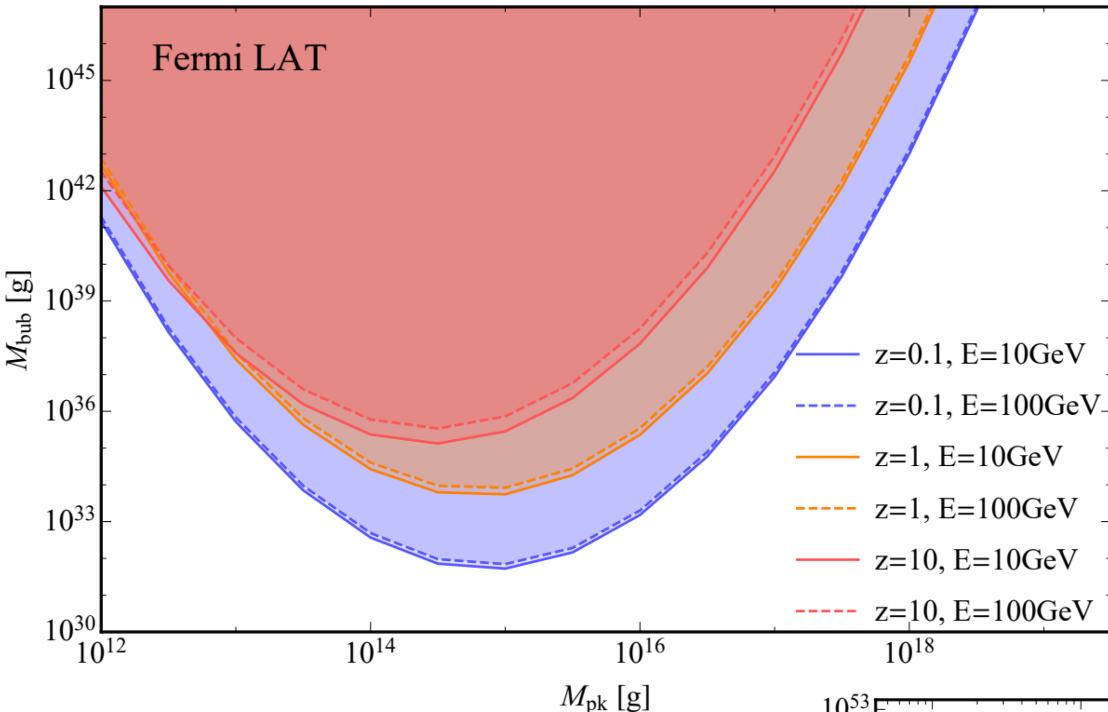
$$n(M;t) = \frac{dN}{dM} = \frac{dN}{dM_i}\frac{dM_i}{dM} = n(M;t_i)\frac{dM_i}{dM}$$

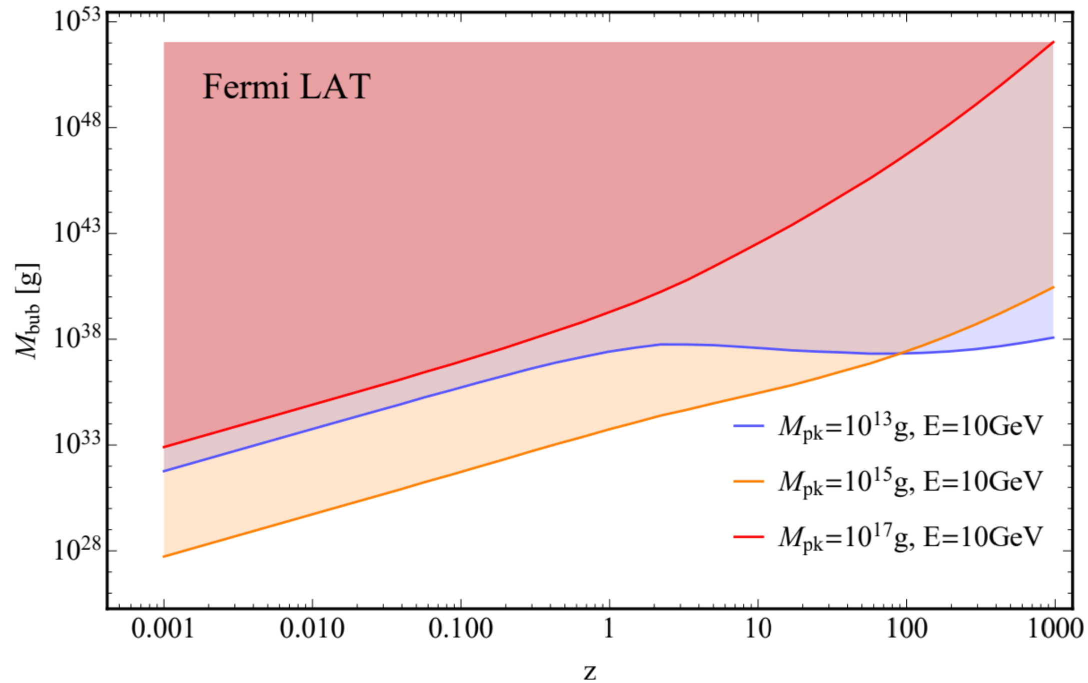
$$\frac{dM}{dt} = -\frac{\alpha}{M^2} \Longrightarrow M^3 = M_i^3 - \delta^3(\Delta t)$$

$$n(M;t) = n(M;t_i)\frac{dM_i}{dM} = n(M;t_i)\frac{M^2}{(M^3 + \delta^3(\Delta t))^{2/3}}$$

$$n(M;t) \simeq \frac{n(\delta(\Delta t);t_i)}{\delta^2(\Delta t)}M^2, \qquad M \ll \delta(\Delta t)$$

Can we see them?





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2105.11481

How to extract the redshift from the observable?

Primary Hawking radiation from the PBH clustering

$$P(E) = \int_0^\infty H_p(E,M)n(M)dM\,,$$

$$H_p(E,M) = \frac{1}{2\pi}\frac{\Gamma_1(E,M)}{e^{8\pi GME}-1} \qquad \Gamma_1(E,M) \propto \begin{cases} G^4M^4E^4, E < (8\pi GM)^{-1} \\ G^2M^2E^2, E > (8\pi GM)^{-1} \end{cases}$$

Redshift in the observed photon flux

$$F(E;z) = \frac{L(E(1+z);z)}{4\pi d_L^2(z)} \simeq \frac{(1+z)^2E^2V}{4\pi d_L^2(z)}\int_0^\infty H_p(E(1+z),M)n(M;z)dM$$

$$H_p(E(1+z),M) = H_p(E,M(1+z))$$

$$\frac{4\pi F(E;z)}{E^2} \simeq \int_0^\infty H_p(E,M')n\left(\frac{M'}{1+z};z\right)\frac{(1+z)V}{d_L^2(z)}dM'$$

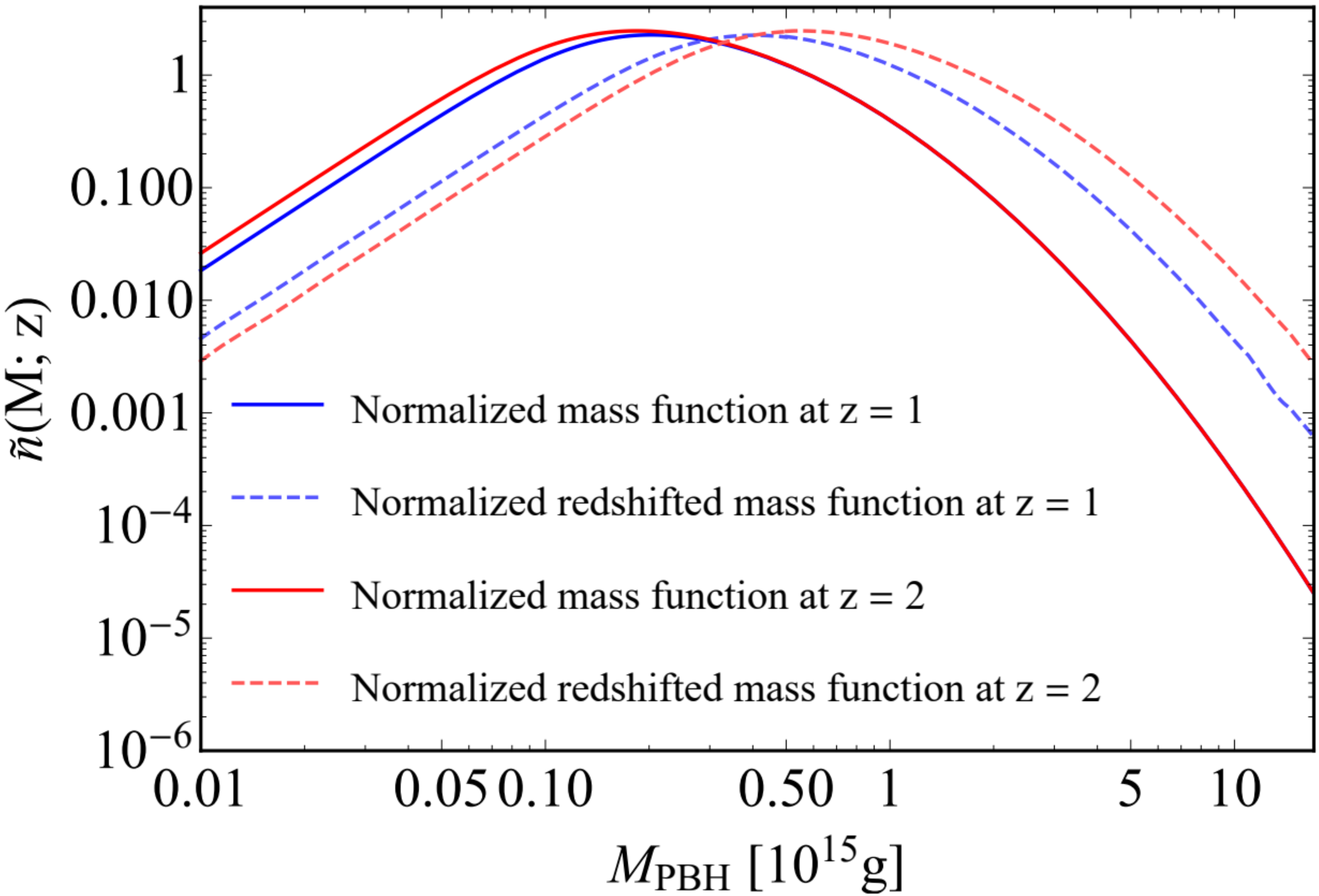
Redshift from the inverse problem

$$P(E) = \int_0^\infty K(E,M) f(M) dM \Longrightarrow f(M) = \int_0^\infty K^{-1}(E,M) P(E) dE$$

$$\frac{4\pi F(E;z)}{E^2} \simeq \int_0^\infty H_p(E,M) n\left(\frac{M}{1+z};z\right)\frac{(1+z)V}{d_L^2(z)} dM$$

$$f(M) \simeq \int_0^\infty H_p^{-1}(E,M) \frac{4\pi F(E;z)}{E^2} dE$$

$$f(M) = n\left(\frac{M}{1+z};z\right)\frac{(1+z)V}{d_L^2(z)}$$

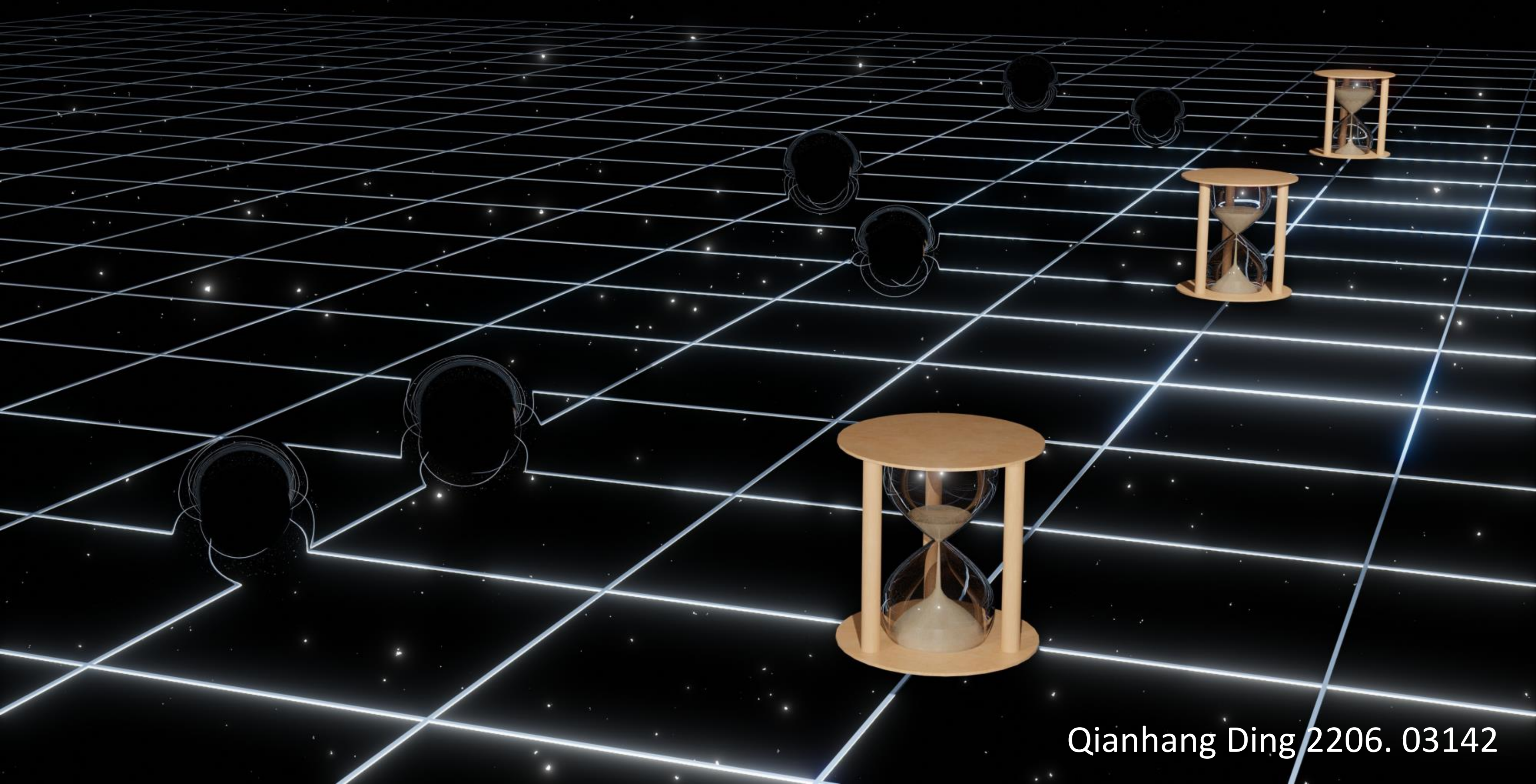


$$n(M) = \frac{f_{PBH}}{\sqrt{2\pi}\sigma M}\exp\left[-\frac{\ln^2(M/M_{pk})}{2\sigma^2}\right] \qquad \tilde{n}(M;z) = n\left(\frac{M}{1+z};z\right)$$

Standard Timers from Primordial Black Hole Binaries

The initial probability distribution on a and e

$$\frac{dP}{dade} = \frac{3}{4} f_{\mathrm{PBH}}^{3/2} \frac{a^{1/2}}{\bar{x}^{3/2}} \frac{e}{(1-e^2)^{3/2}}$$



Qianhang Ding 2206. 03142

How to extract the physical evolution time?

The evolution of probability distribution in PBH binaries

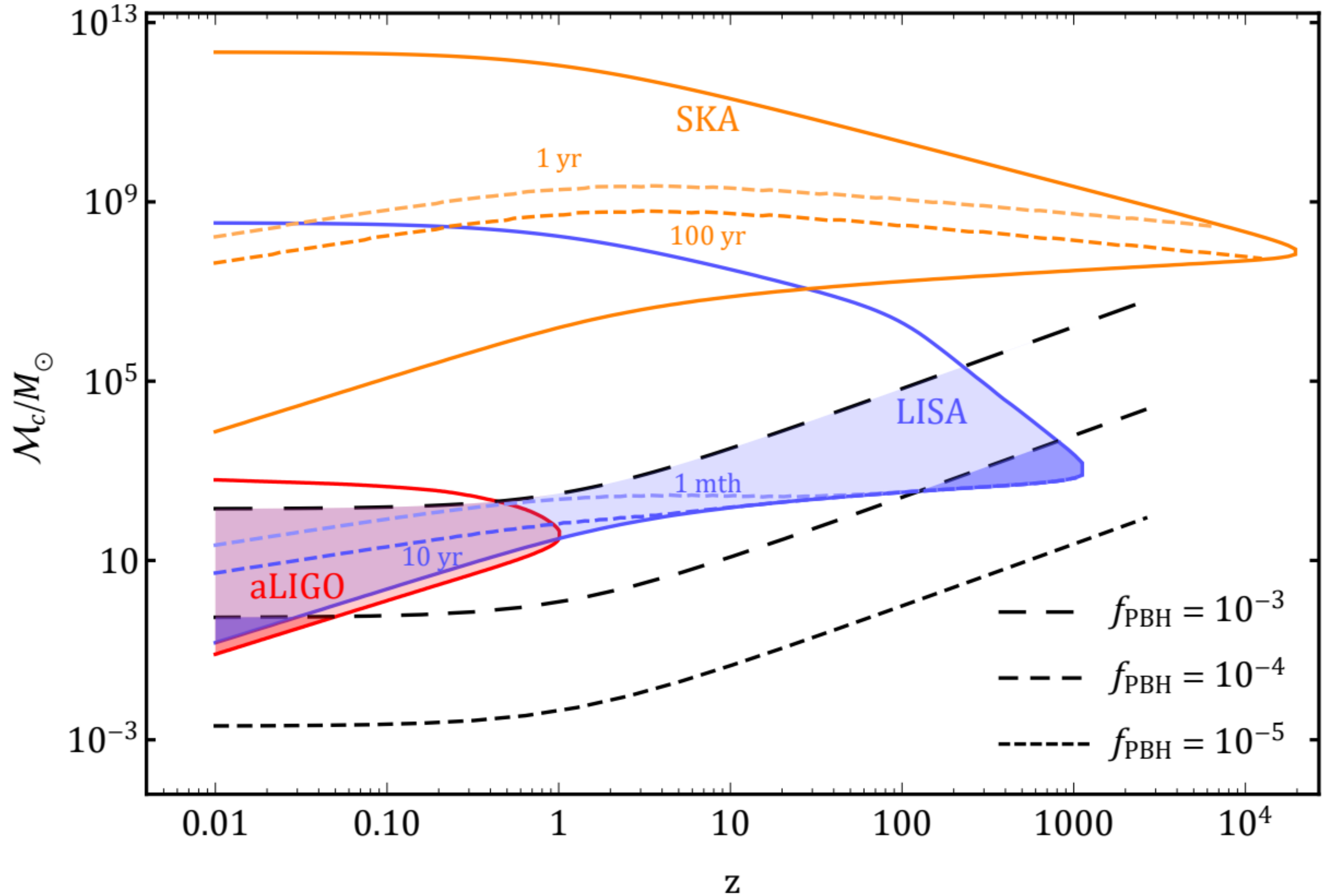
$$\frac{dP}{da_t de_t} = \frac{dP}{da_i de_i} \det \mathrm{J}(a, e, \Delta t)$$

$$\mathrm{J}(a, e, \Delta t) = \begin{pmatrix} \partial a_i / \partial a_t & \partial a_i / \partial e_t \\ \partial e_i / \partial a_t & \partial e_i / \partial e_t \end{pmatrix}$$

$$\frac{da}{dt} = -\frac{128}{5} \frac{G^3 M_{\mathrm{PBH}}^3}{c^5 a^3 (1 - e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right)$$

$$\frac{de}{dt} = -\frac{608}{15} \frac{G^3 M_{\mathrm{PBH}}^3}{c^5 a^4} \frac{e}{(1 - e^2)^{5/2}} \left(1 + \frac{121}{304} e^2 \right)$$

Can we see them?



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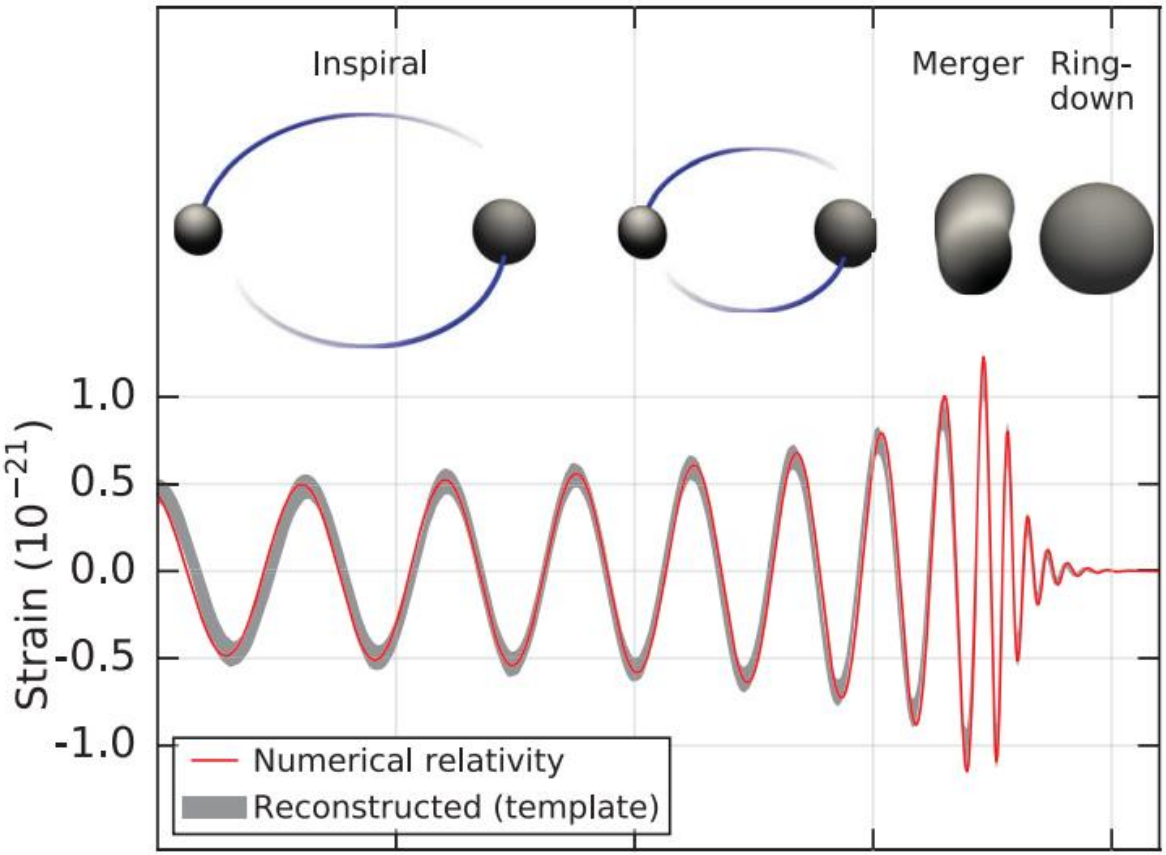
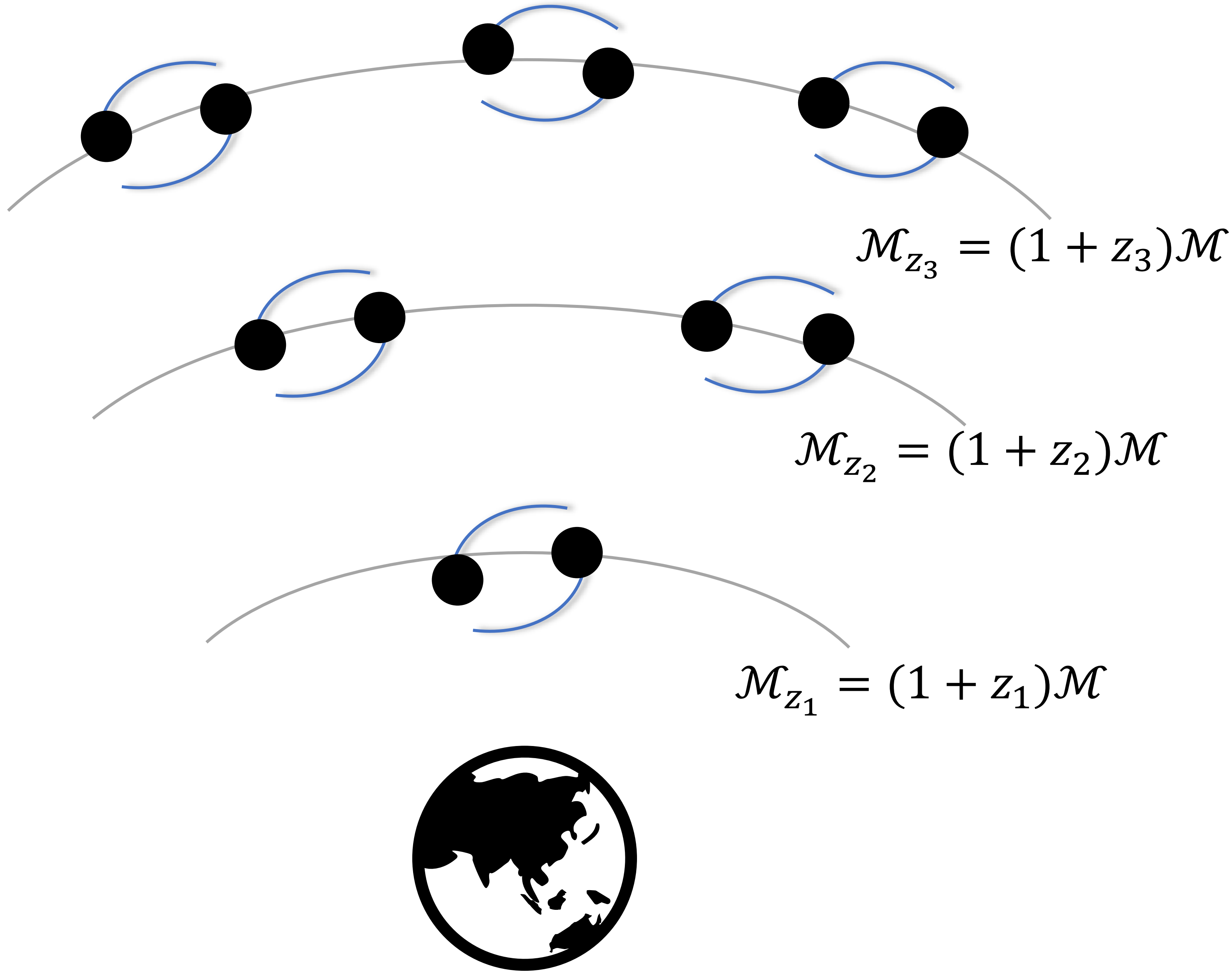
How to extract the redshift from the observable?

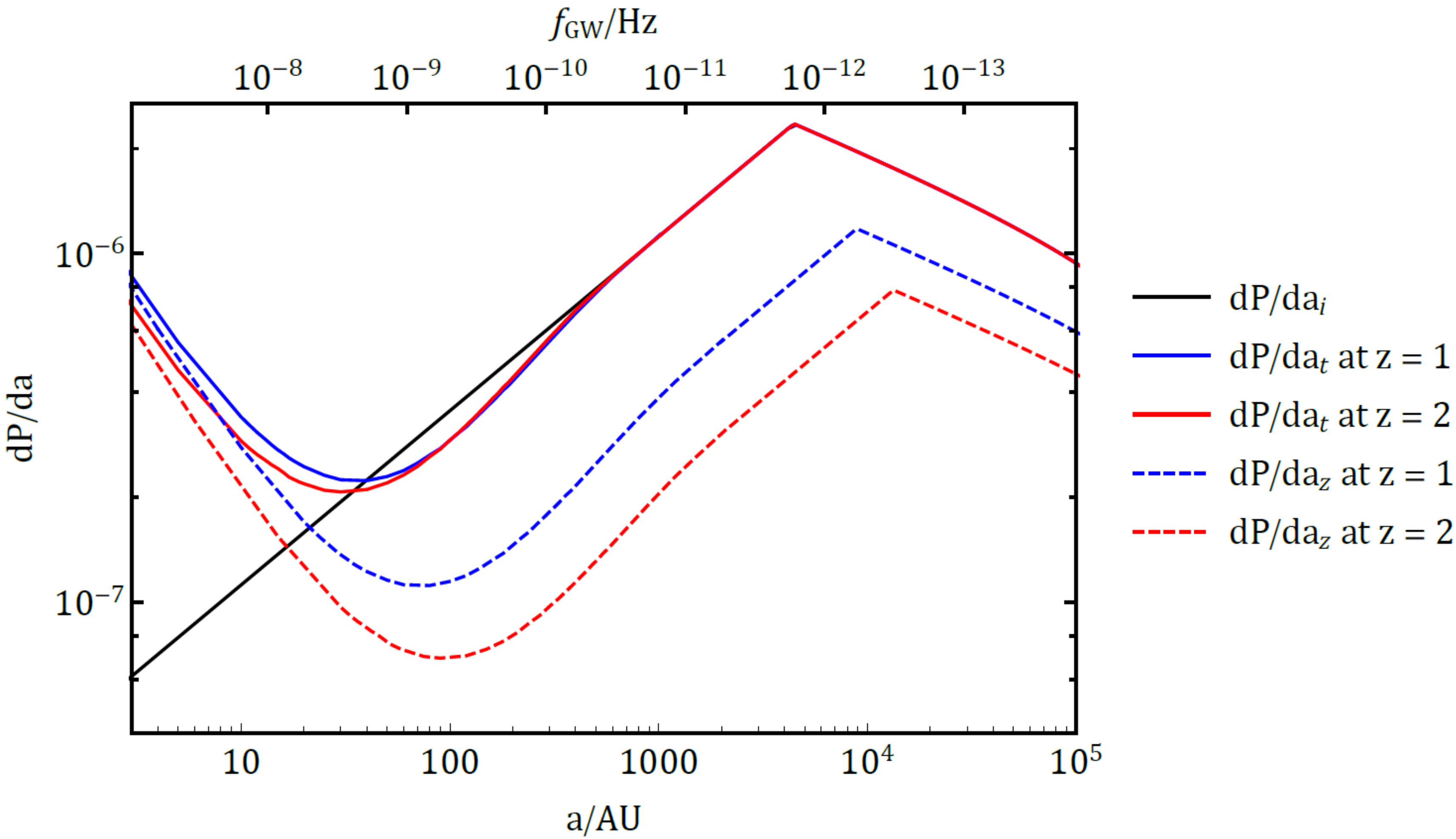


B. P. Abbott et al. Observation of Gravitational Waves from a Binary Black Hole Merger. Phys. Rev. Lett., 116(6):061102, 2016.

Redshifted Chirp Mass

$$\mathcal{M}_z = (1 + z)\mathcal{M}$$

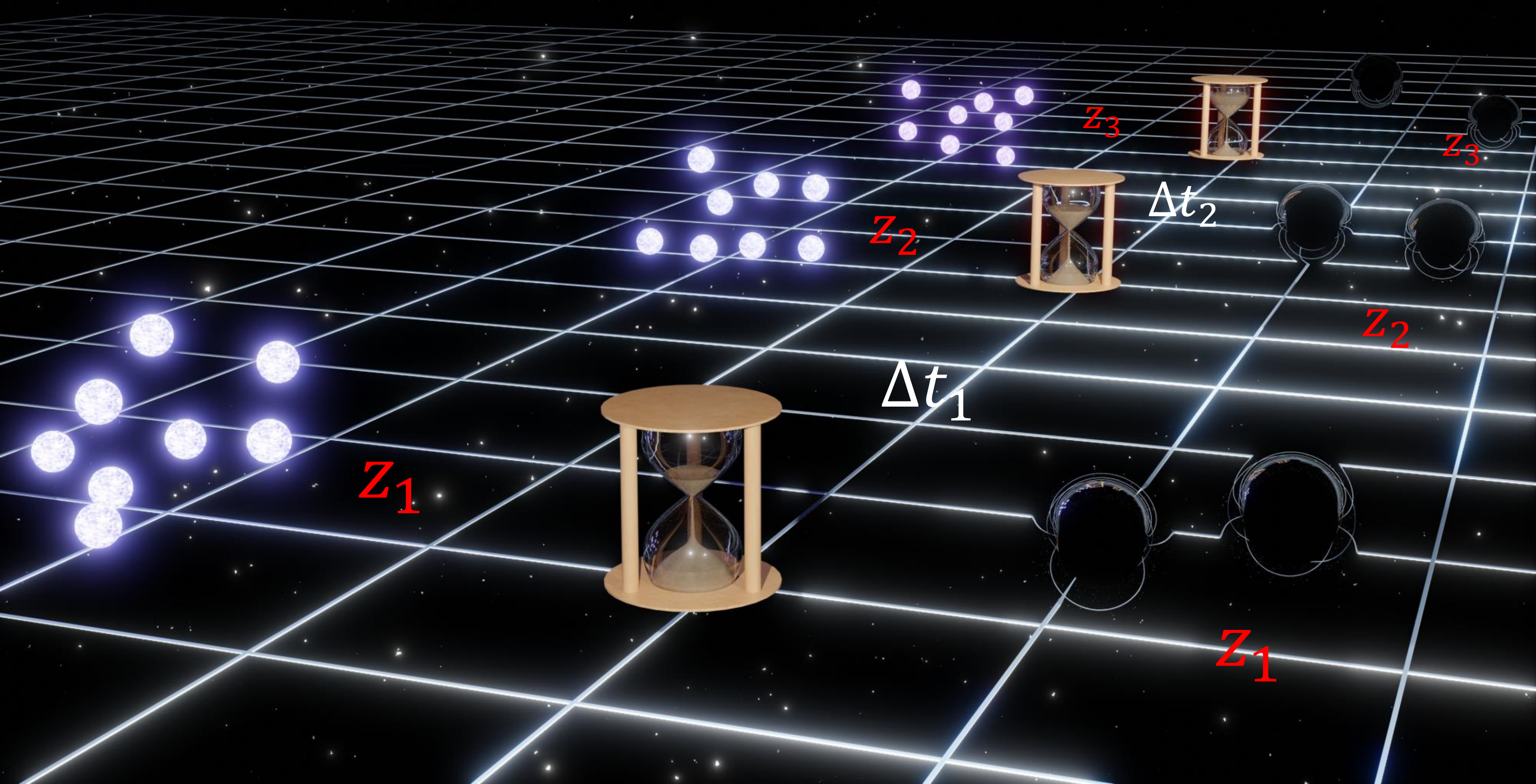

$\mathcal{M}_{z_3} = (1+z_3)\mathcal{M}$
$\mathcal{M}_{z_2} = (1+z_2)\mathcal{M}$
$\mathcal{M}_{z_1} = (1+z_1)\mathcal{M}$



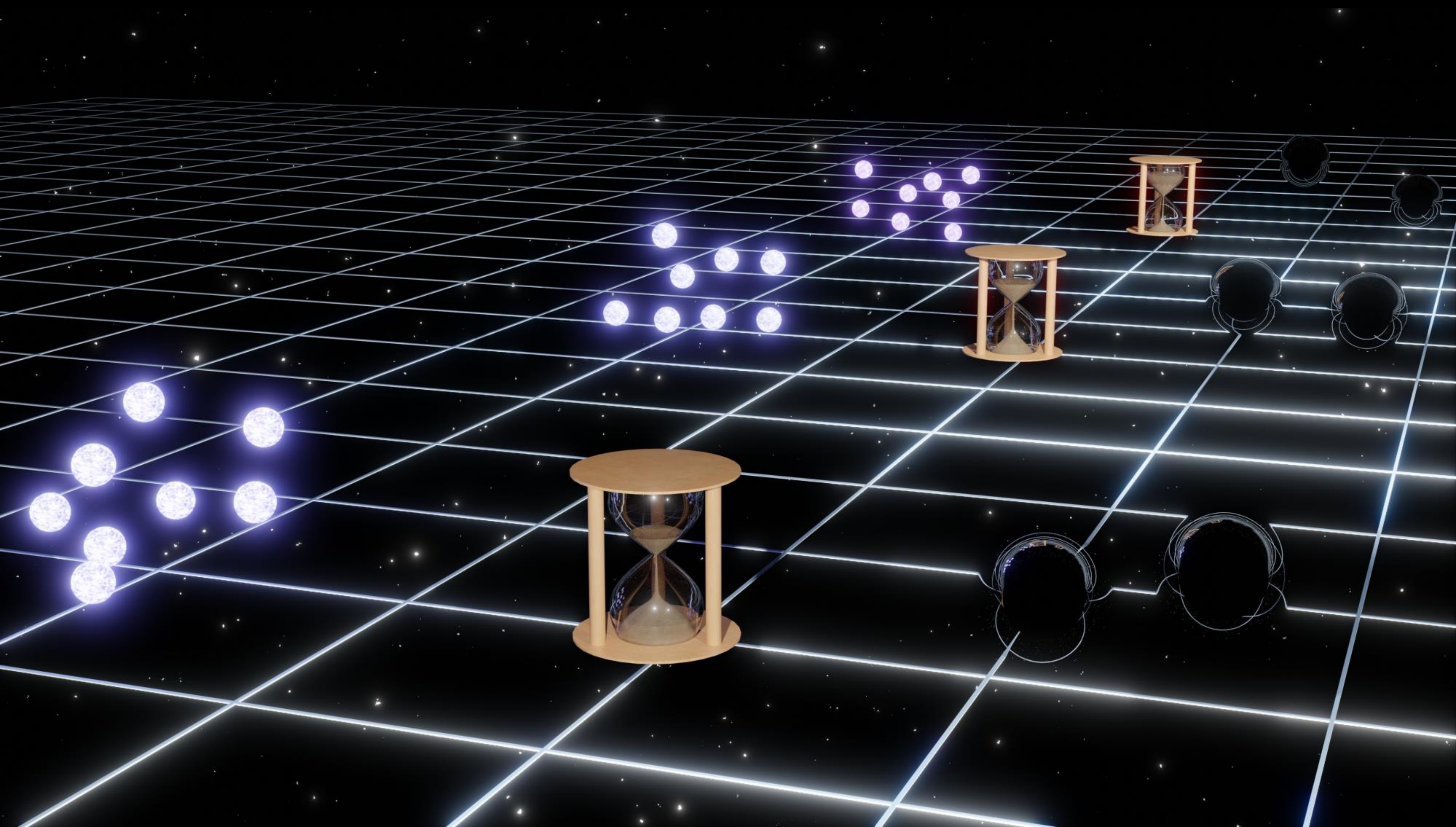
$$\frac{dP}{da_z} = \frac{1}{1+z}\frac{dP}{da_i}$$

$$\Delta t = \int_{z_1}^{z_2} \frac{dz}{H(z)(1+z)}$$

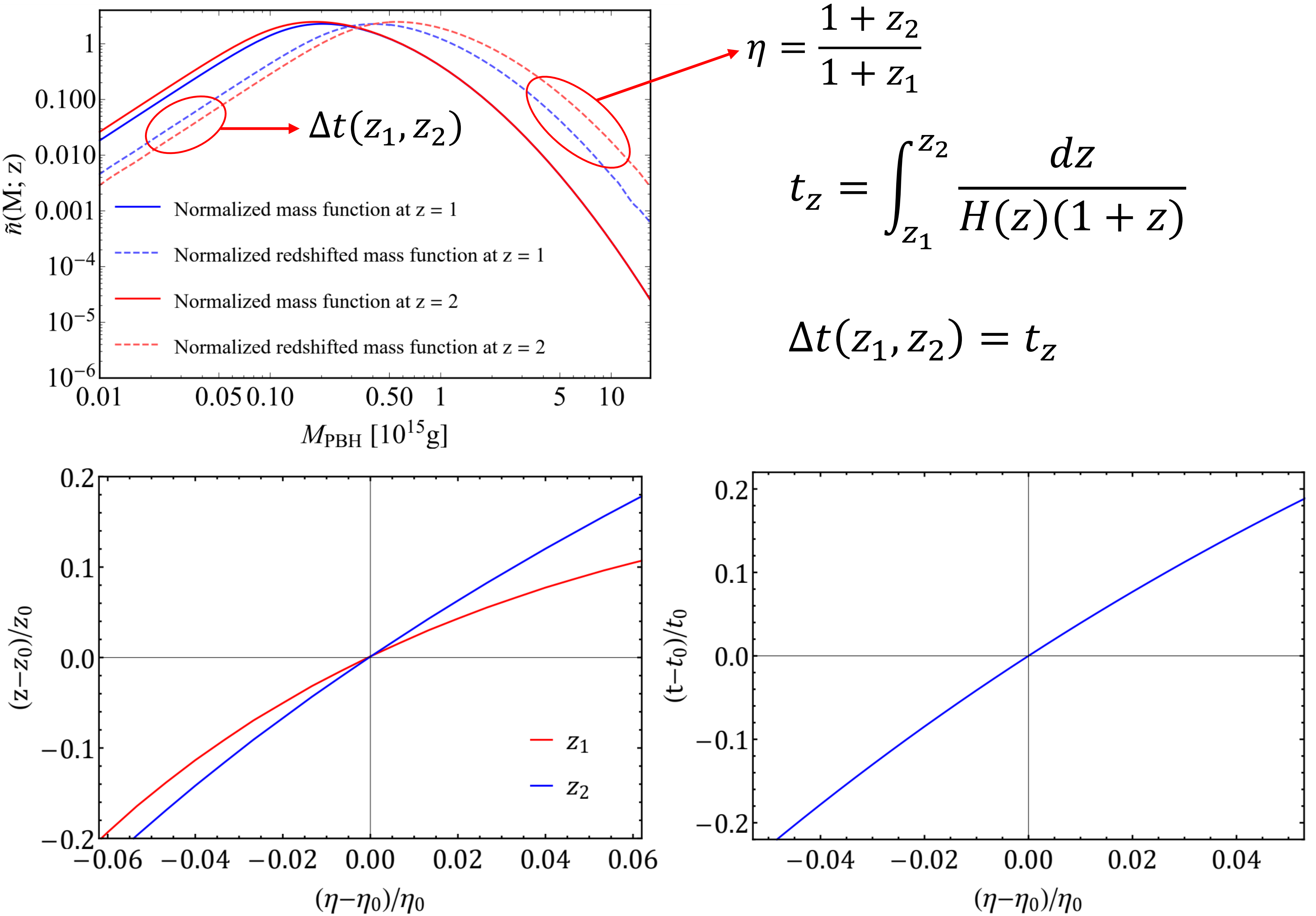
$$H(z) = H_0\sqrt{\Omega_\gamma(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda}$$



Thank you !

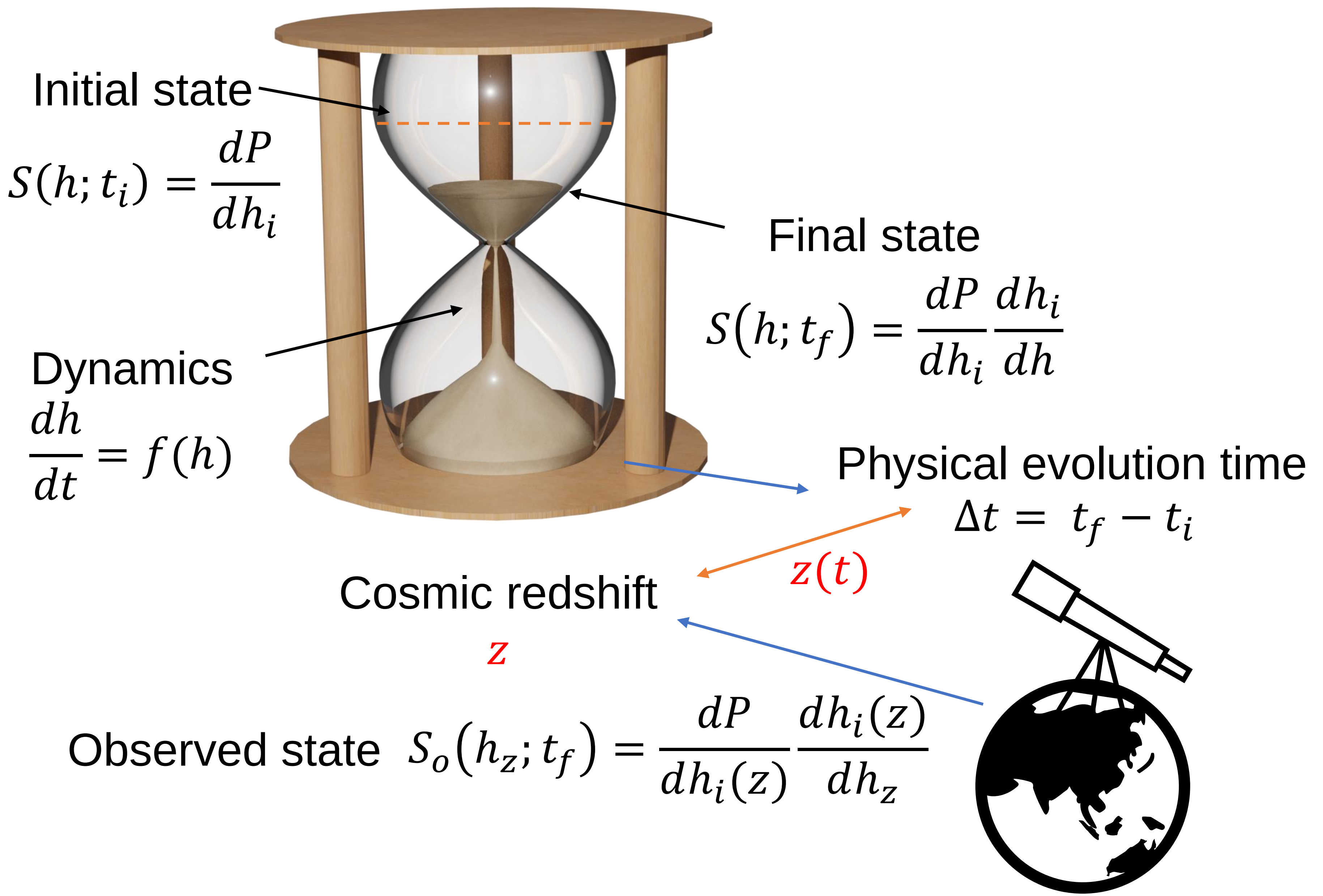



Thank you and welcome to visit HKUST



$\Delta t(z_1, z_2)$
$\eta = \frac{1+z_2}{1+z_1}$
$t_z = \int_{z_1}^{z_2} \frac{dz}{H(z)(1+z)}$
$\Delta t(z_1, z_2) = t_z$
$\tilde{n}(M; z)$
$M_{\rm PBH}$ [10^{15}g]
Normalized mass function at z = 1
Normalized redshifted mass function at z = 1
Normalized mass function at z = 2
Normalized redshifted mass function at z = 2
$(z-z_0)/z_0$
$(\eta-\eta_0)/\eta_0$
z_1
z_2
$(t-t_0)/t_0$
$(\eta-\eta_0)/\eta_0$

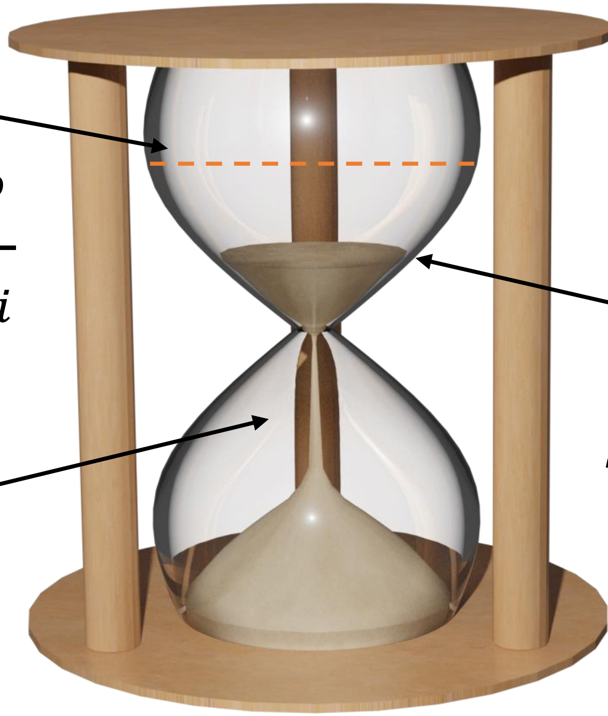
Standard timers in dynamical systems



Standard timers in dynamical systems

Initial state

$$S(h;t_i) = \frac{dP}{dh_i}$$



Final state

$$S(h;t_f) = \frac{dP}{dh_i}\frac{dh_i}{dh}$$

Dynamics

$$\frac{dh}{dt} = f(h)$$

Observed state

$$P(E) = \int K(E,h_z)S_o(h_z;t_f)dh_z$$

$$S_o(h_z;t_f) = \int K^{-1}(E,h_z)P(E)dE$$