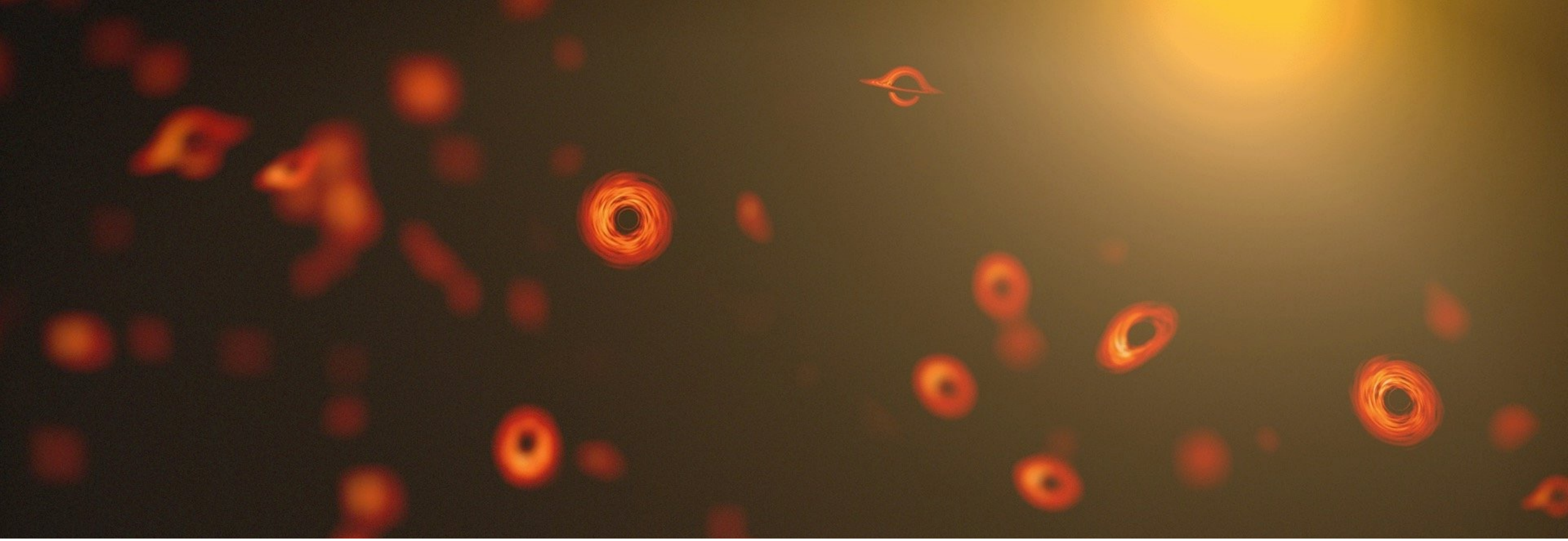


Primordial Black Hole
Future Perspective in Gravitational Wave Cosmology

Qianhang Ding

IBS CTPU-CGA

IBS CTPU-CGA 2025 Workshop for High Energy Physics and Cosmology in Korea

Primordial Black Hole

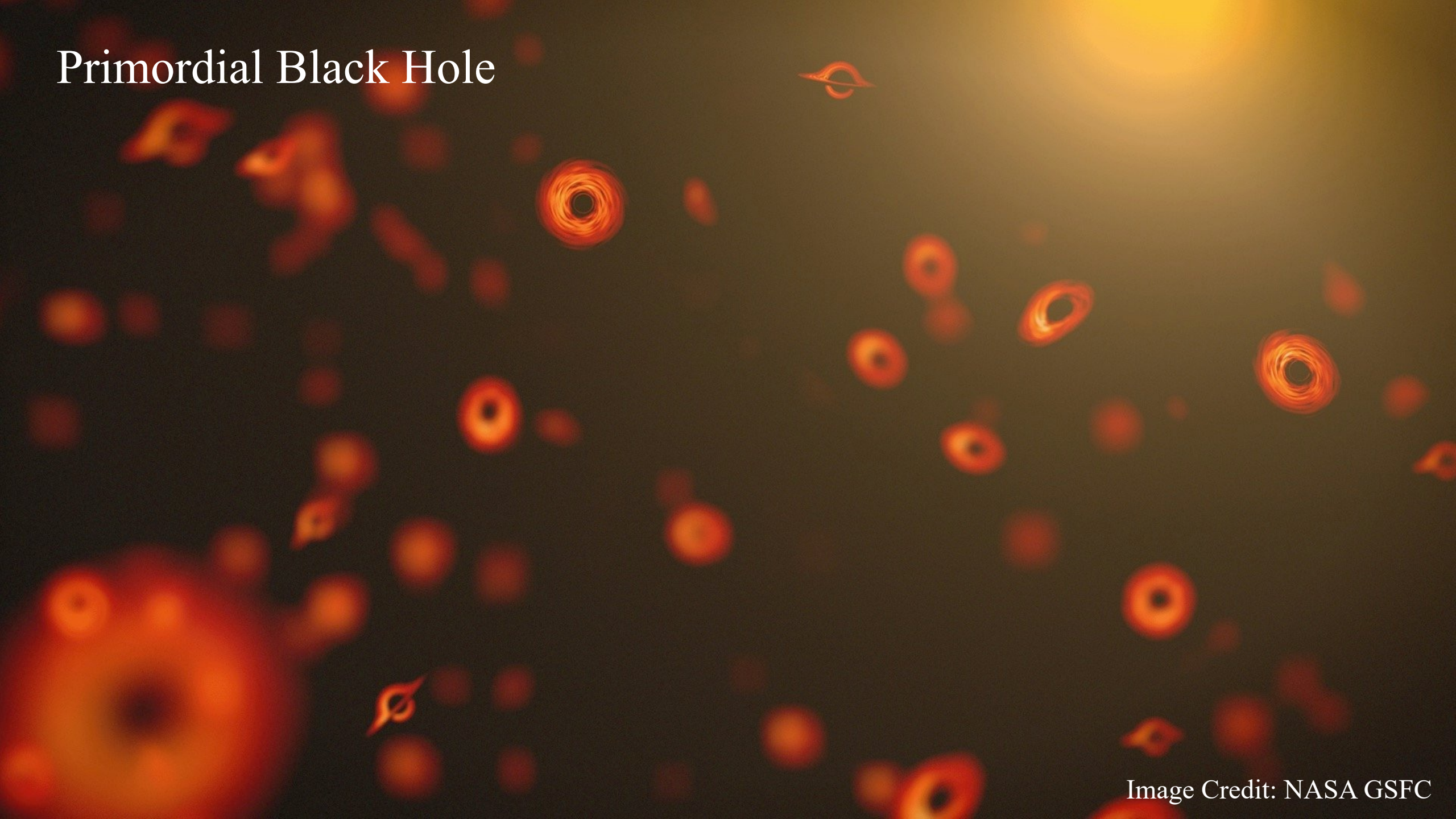


Image Credit: NASA GSFC

Primordial Black Hole

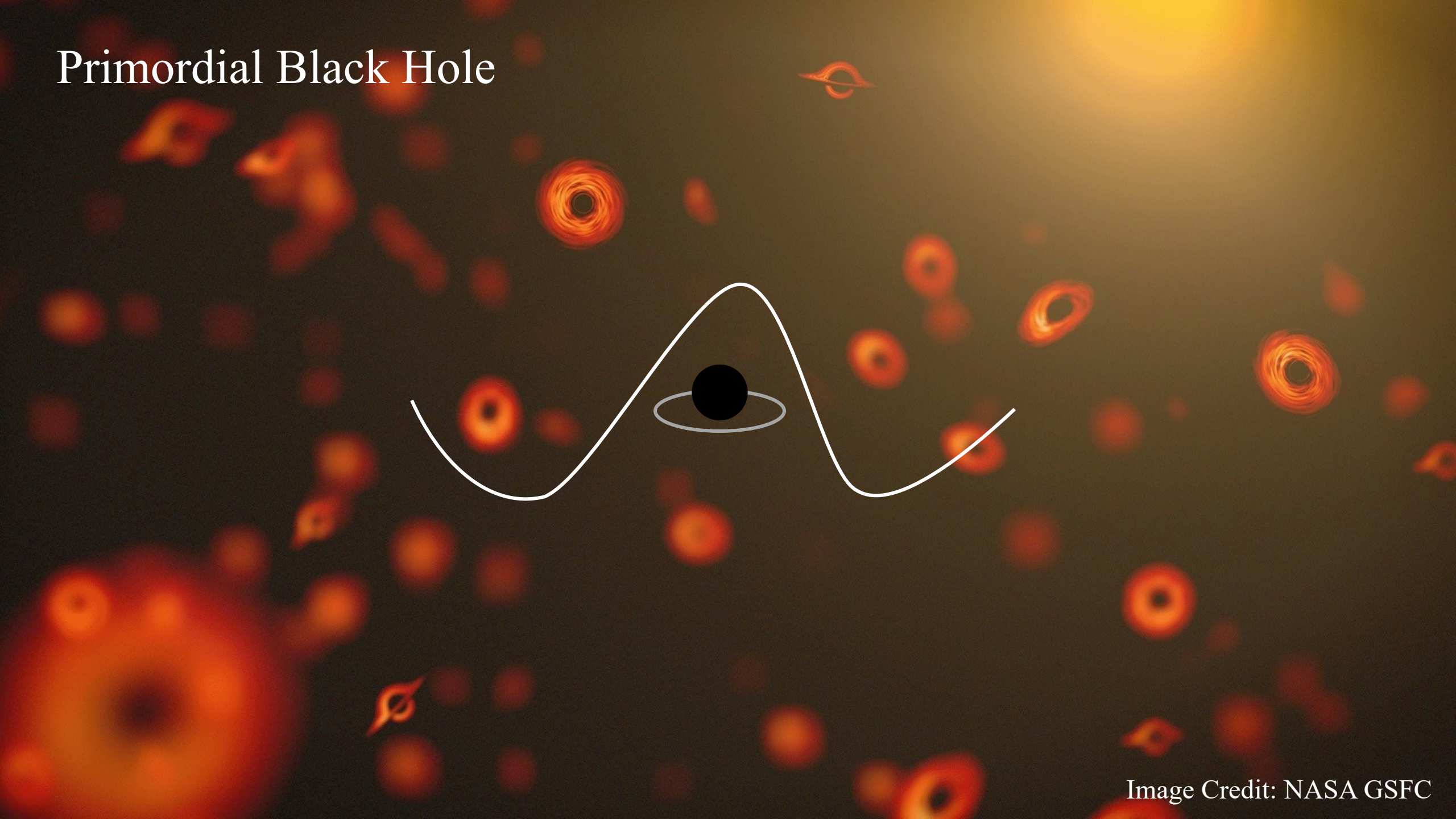


Image Credit: NASA GSFC

Primordial Black Hole

$$M_{PBH} \sim M_H \sim \frac{c^3}{G} t \sim 10^{15} \left(\frac{t}{10^{-23}\ s} \right) g$$

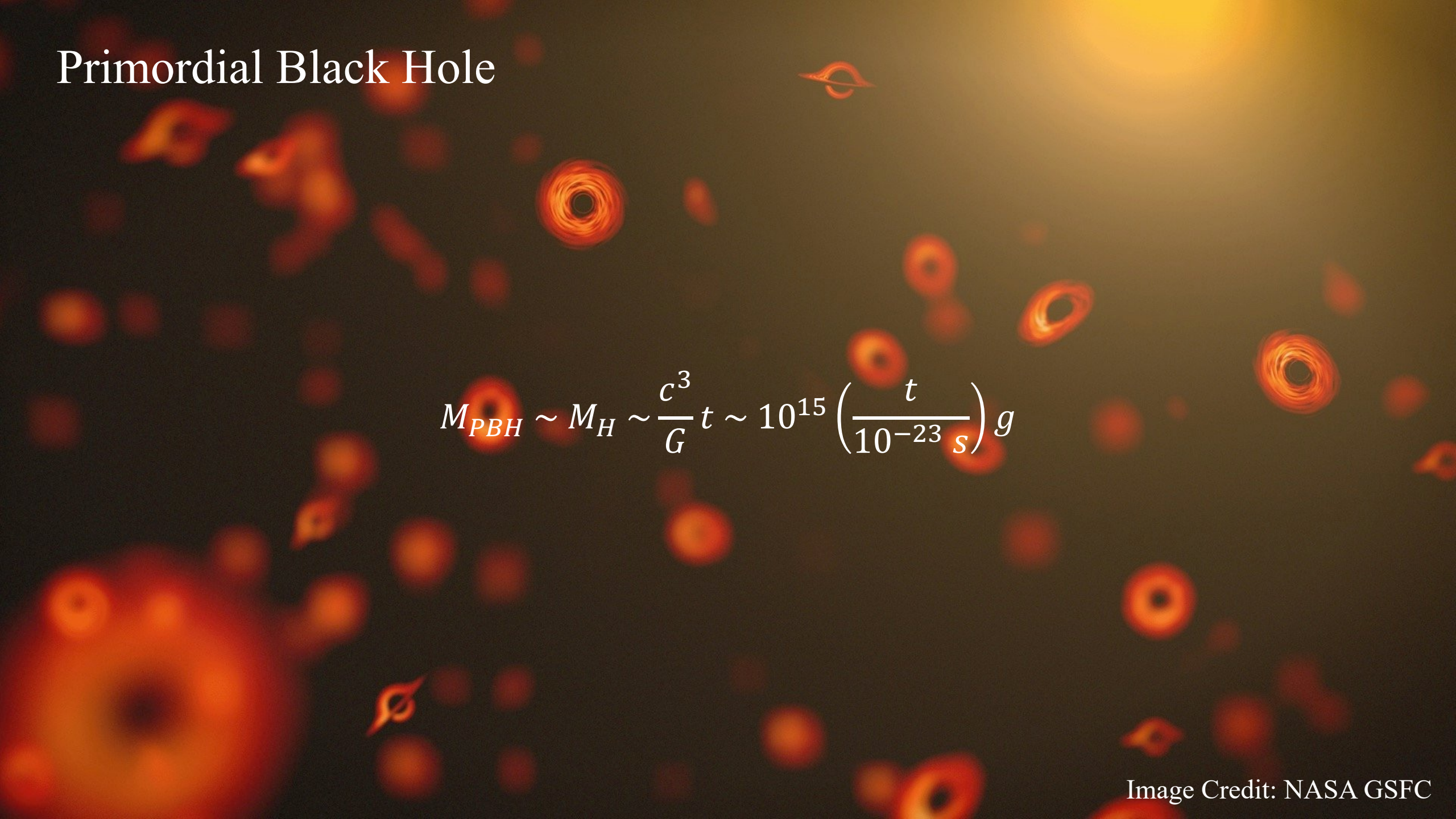
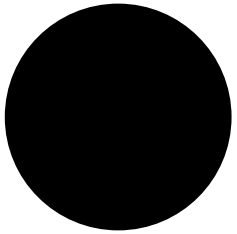


Image Credit: NASA GSFC

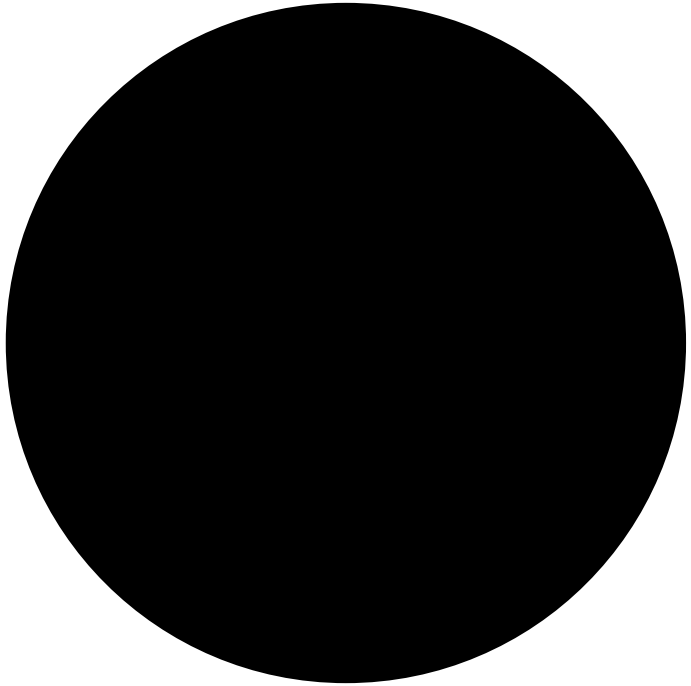
Typical PBH Mass

10^{15}g
Hawking Radiation

5 M_E
Planet 9

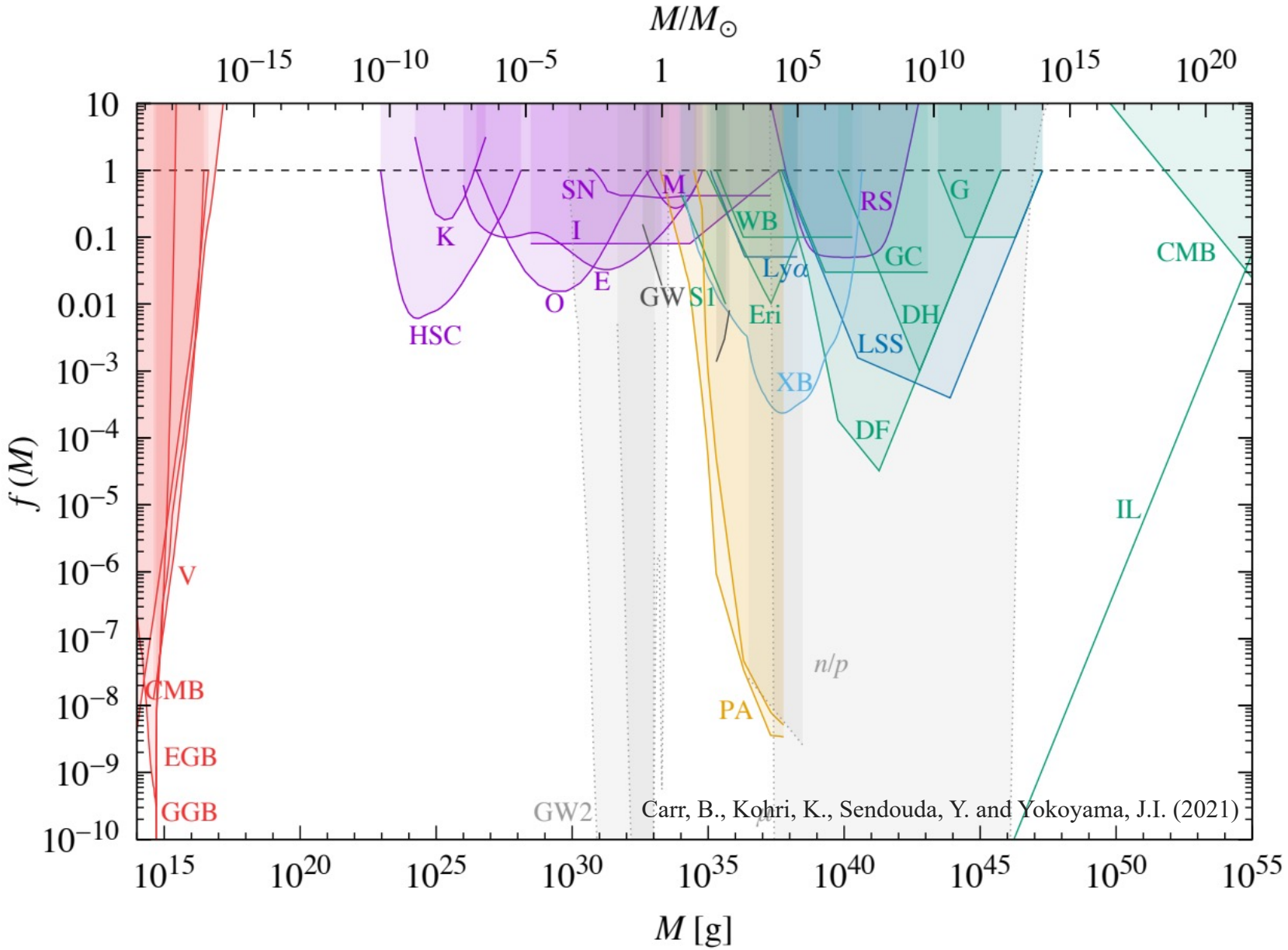


30 $M_\odot$
LVK Events

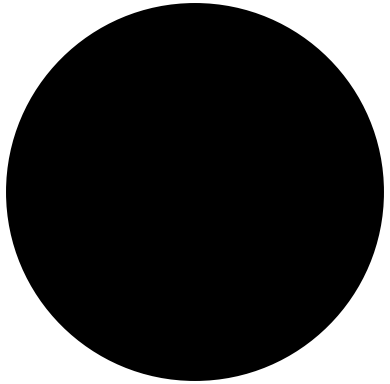


10^5~10^{10} $M_\odot$
SMBH

Primordial Black Hole as Dark Matter



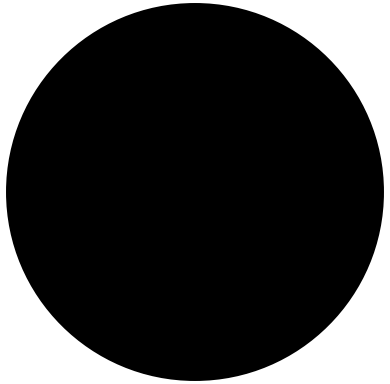
Primordial?



Astrophysical?

Intrinsic Property

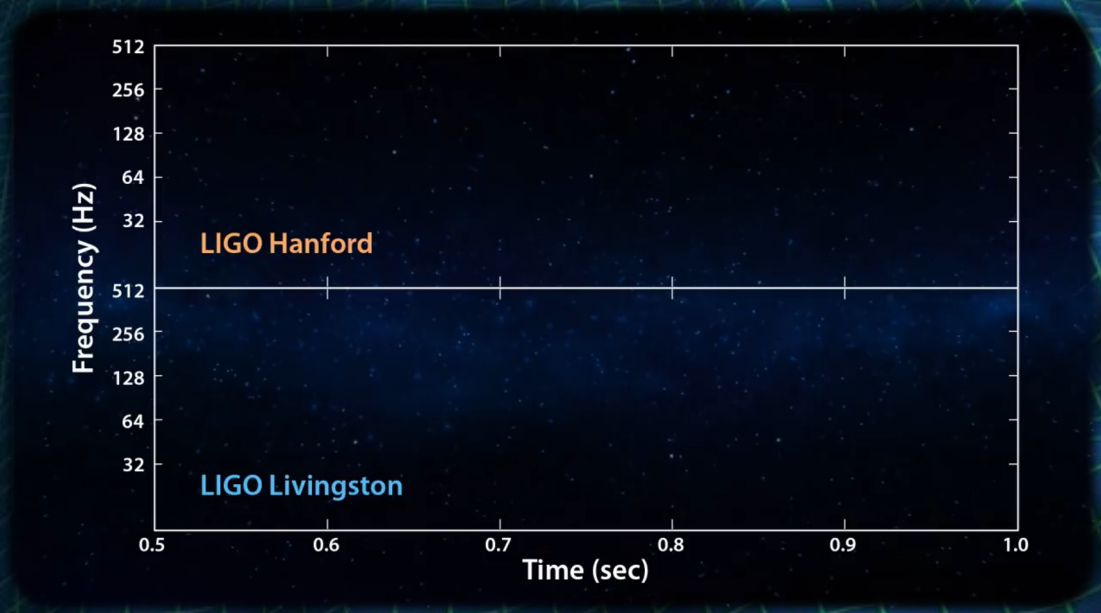
Mass Spin Charge



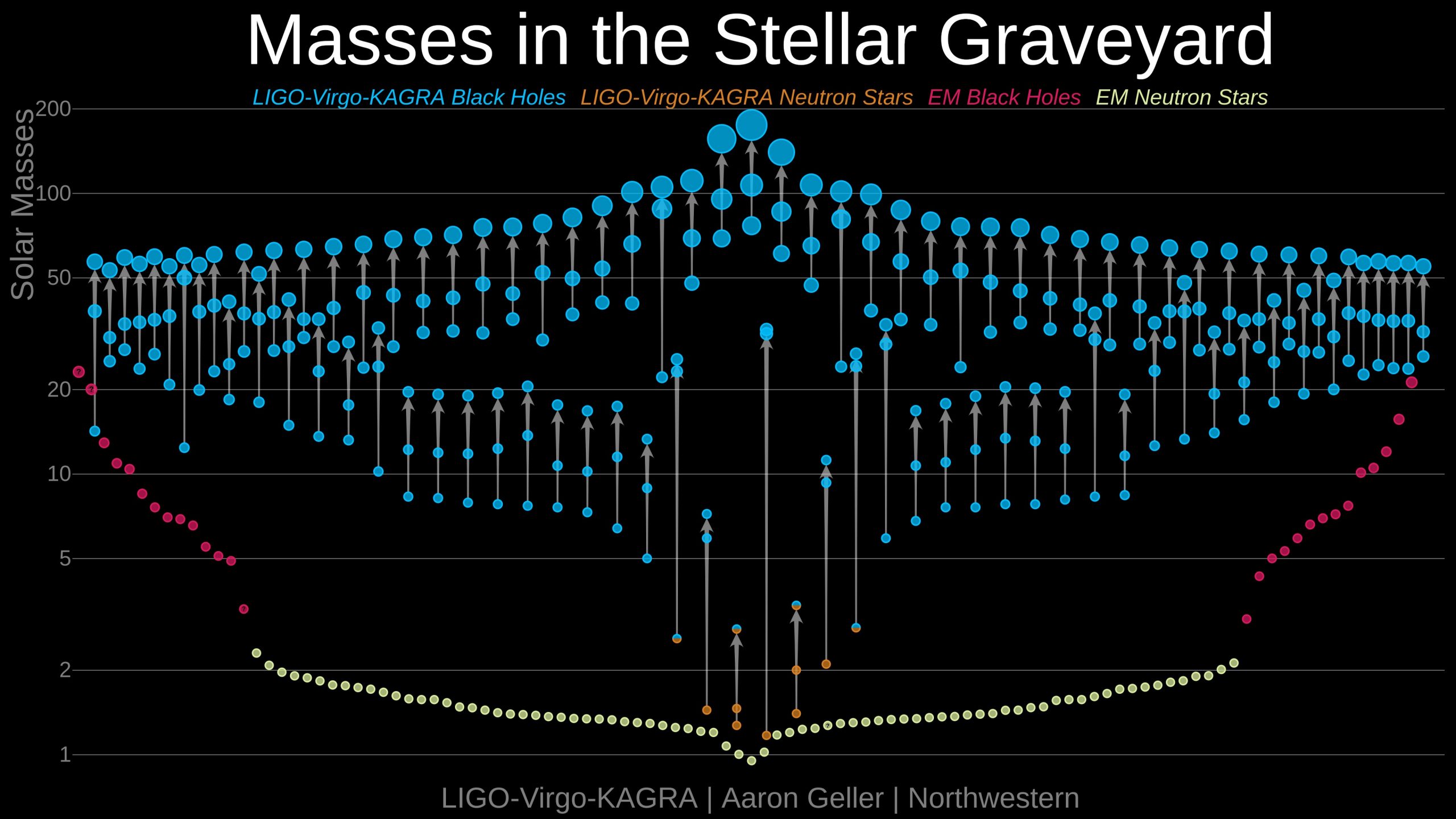
Formation History

Redshift Environment SGWB

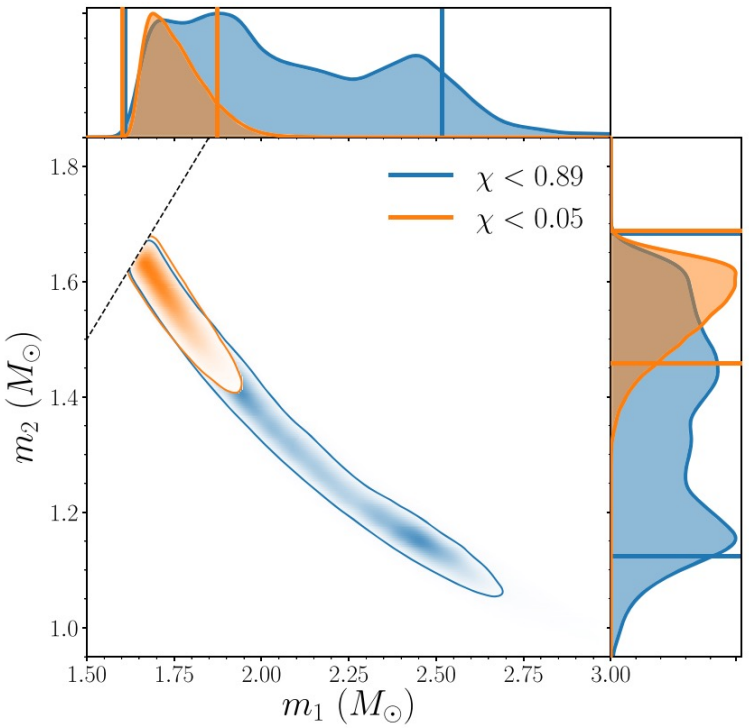
GW150914


512
256
128
64
32
LIGO Hanford
512
256
128
64
32
LIGO Livingston
Frequency (Hz)
0.5
0.6
0.7
0.8
0.9
1.0
Time (sec)

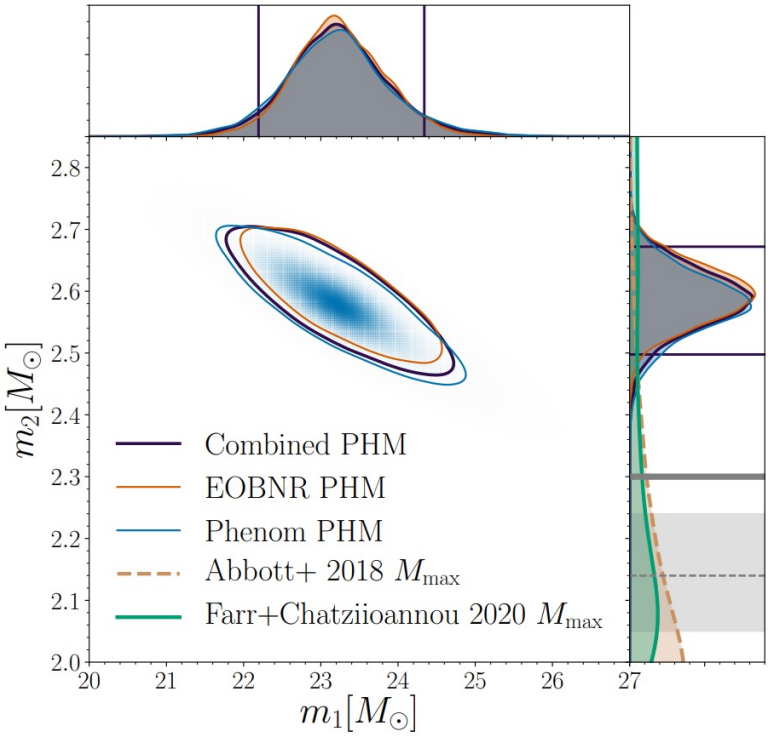
Masses in the Stellar Graveyard



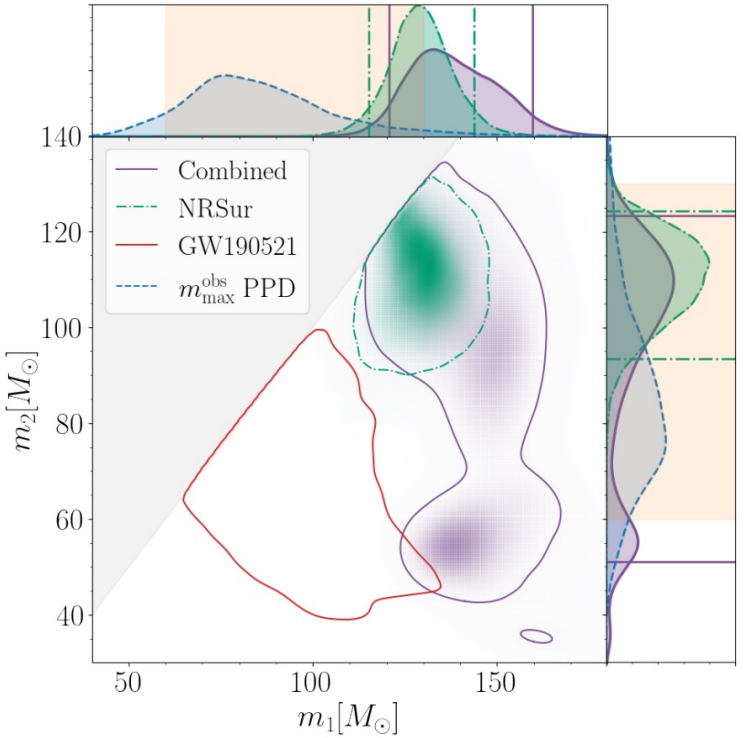
Single Event



GW190425

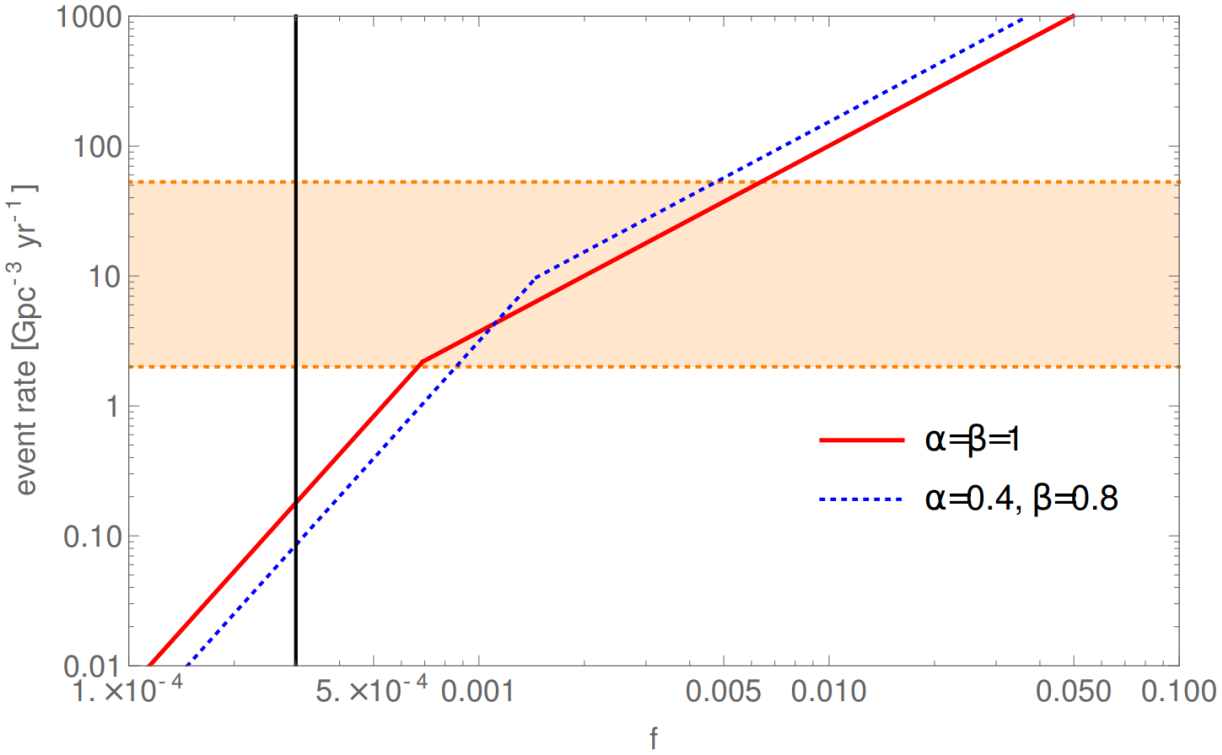


GW190814

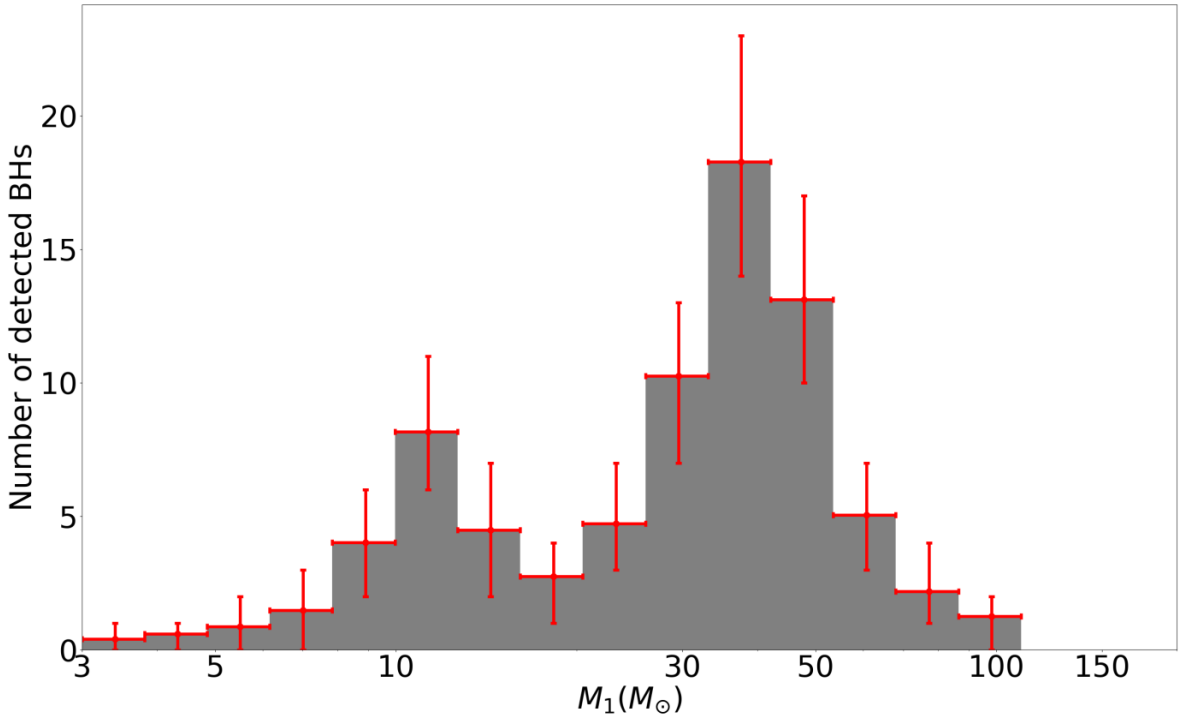


GW231123

Statistic Information



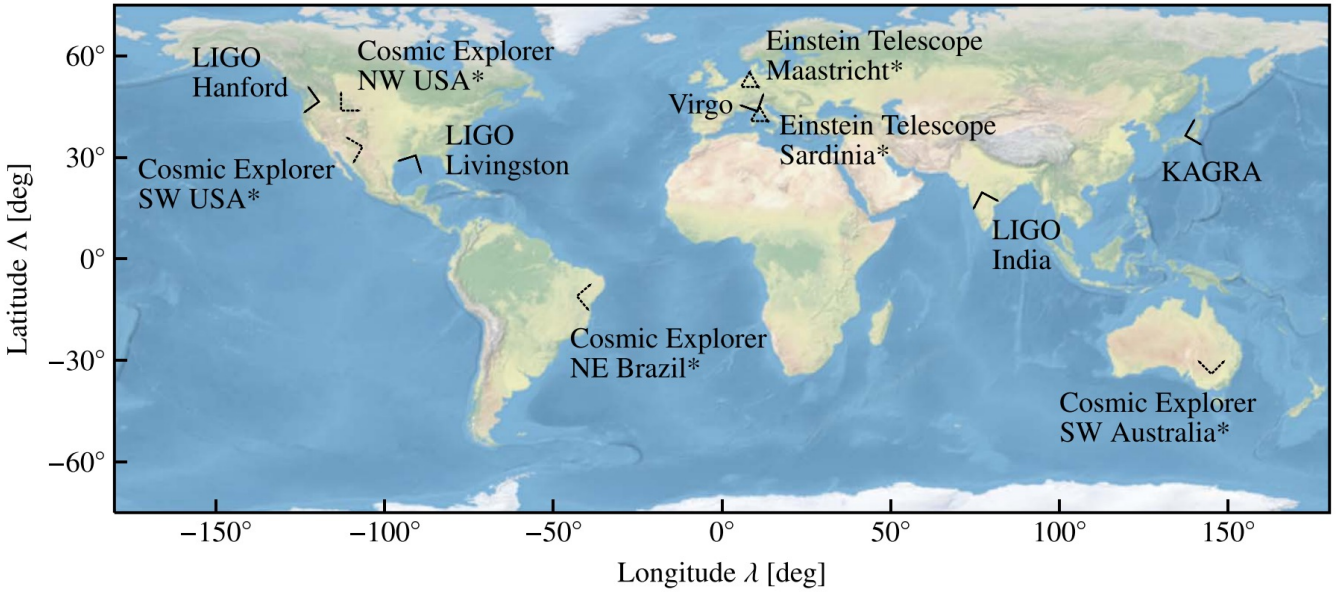
Sasaki, M., Suyama, T., Tanaka, T. and Yokoyama, S. (2016)



El Bouhaddouti, M. and Cholis, I. (2025)

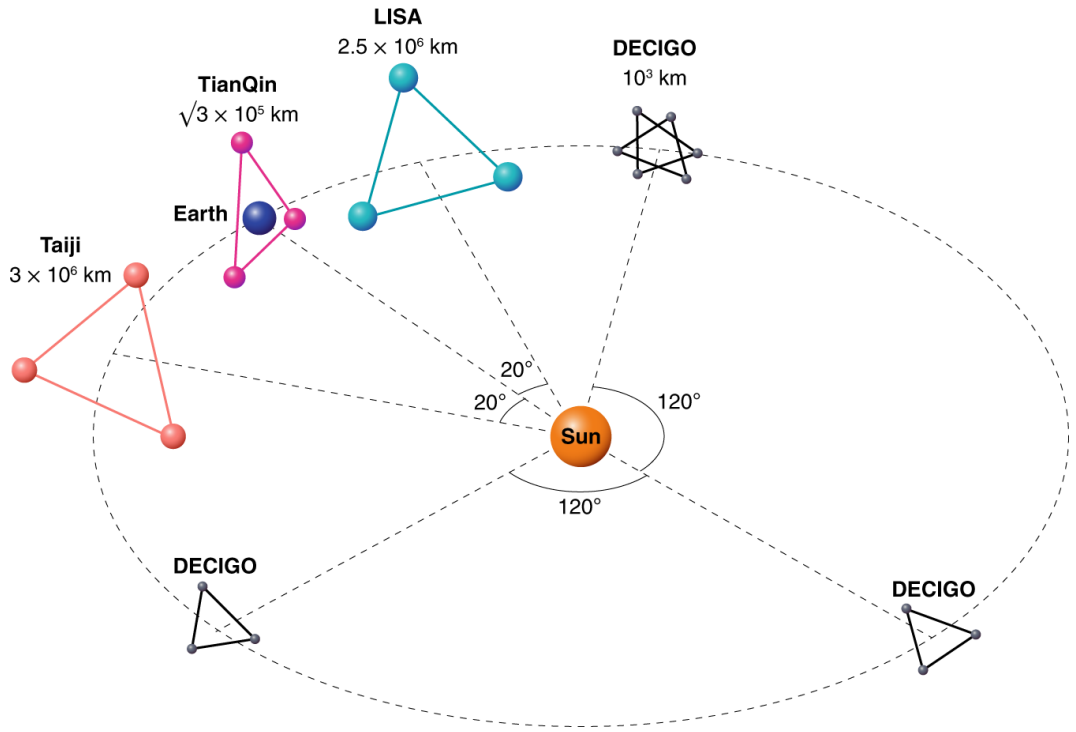
Looking into the future!

Future Perspective



Ground-Based GW Detectors

Gossan, S.E., Hall, E.D. and Nissanke, S.M. (2022)



Space-Based GW Detectors

Gong, Y., Luo, J. and Wang, B. (2021)

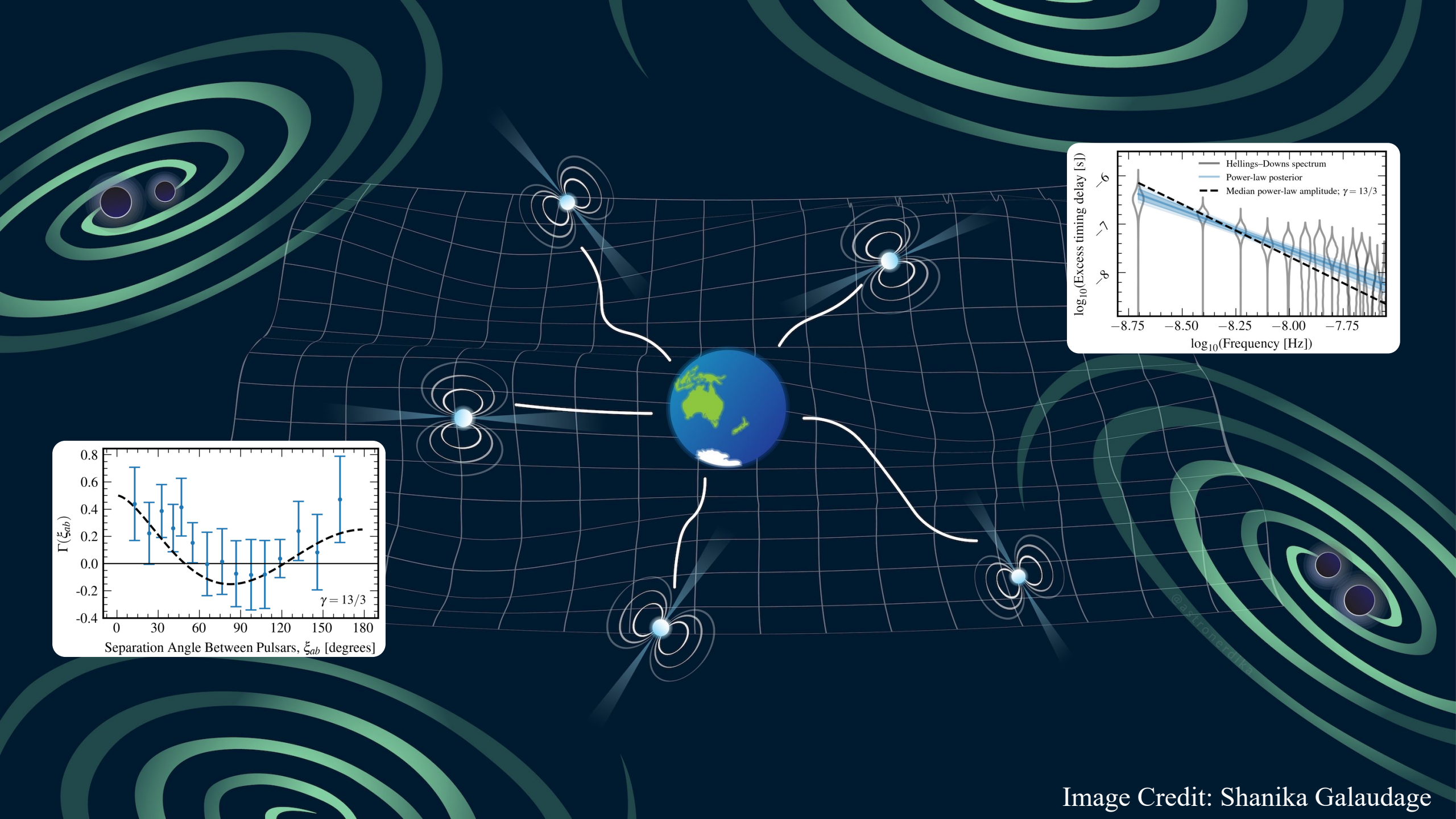
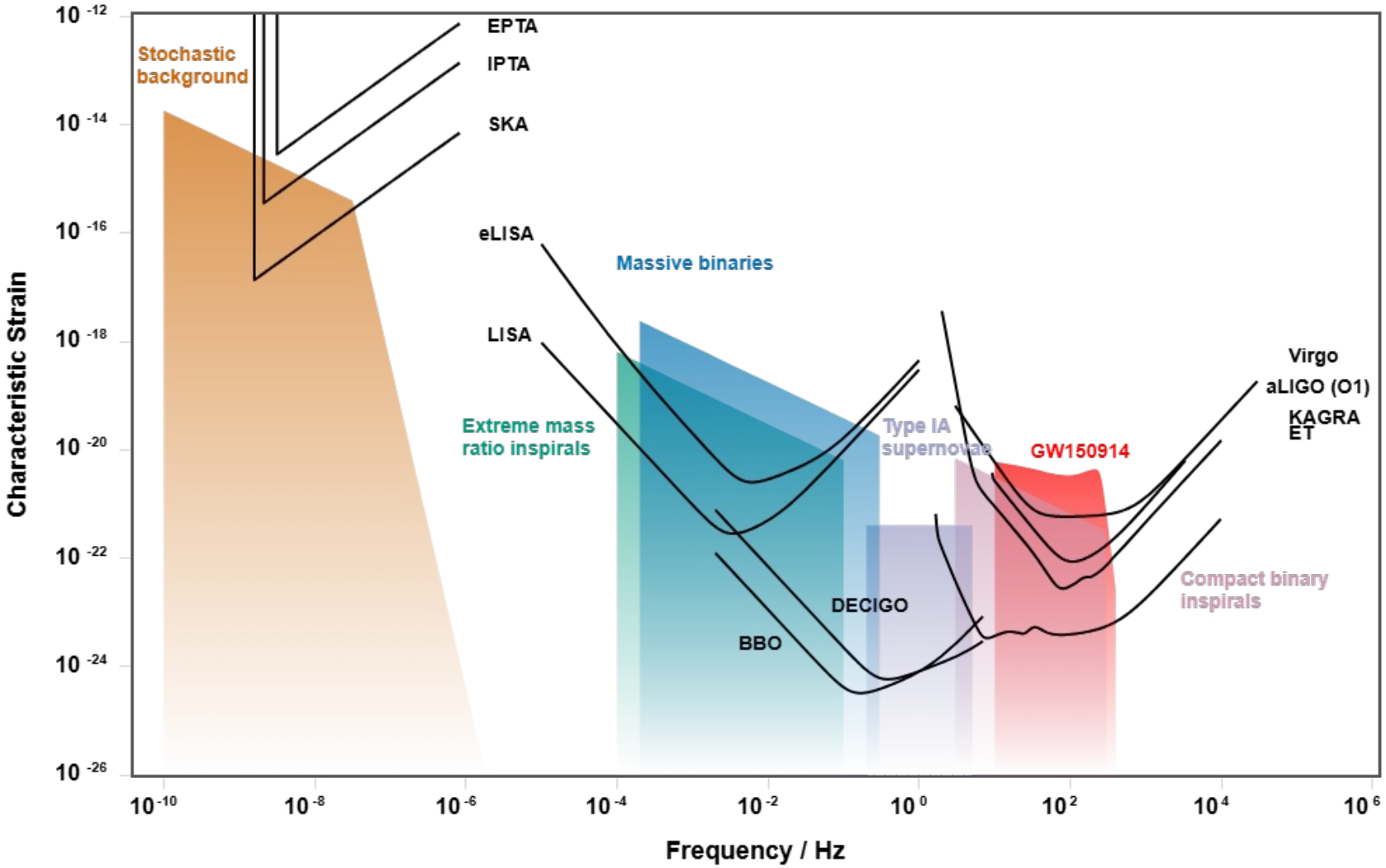


Image Credit: Shanika Galaudage

Image Credit: Olena Shmahalo

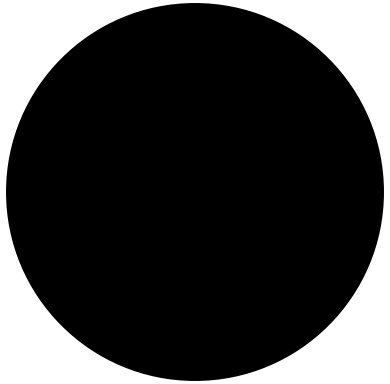
Future Perspective



Moore, C.J., Cole, R.H. and Berry, C.P. (2014)

Intrinsic Property

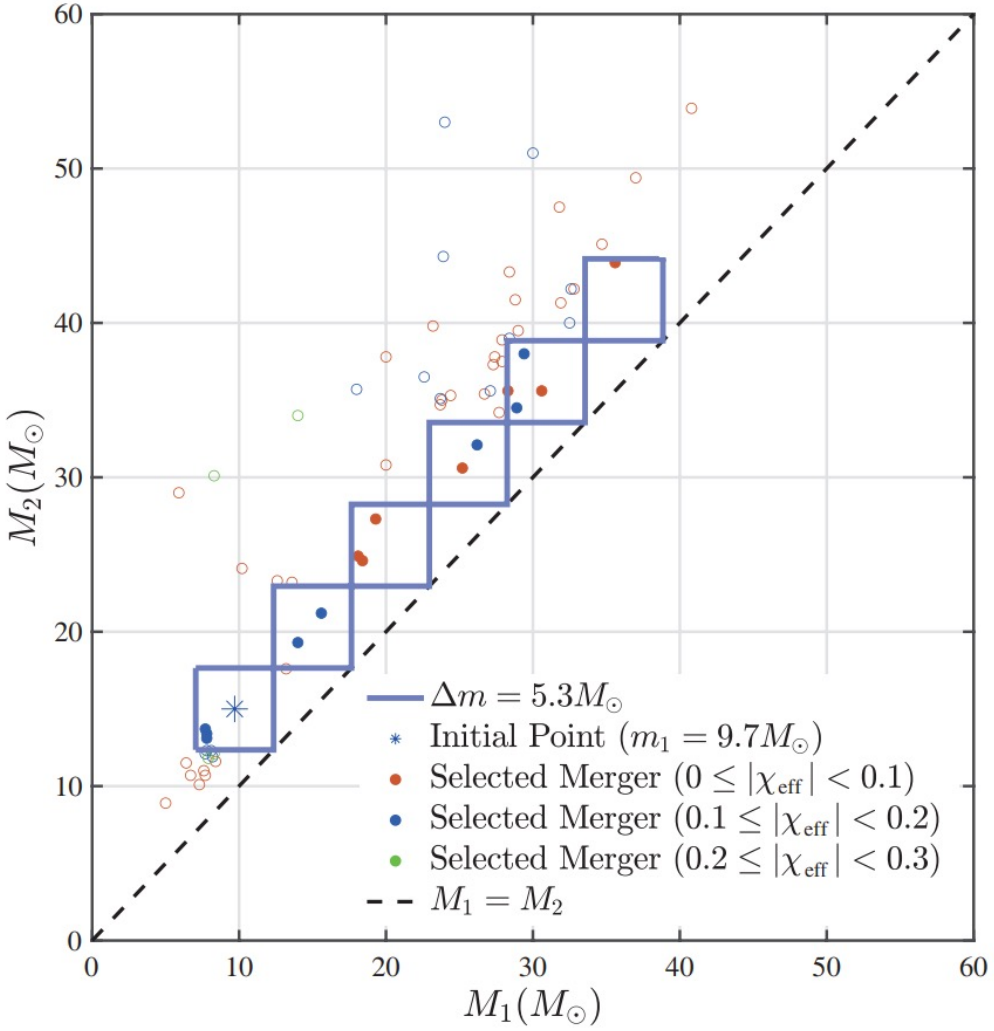
Mass Spin Charge

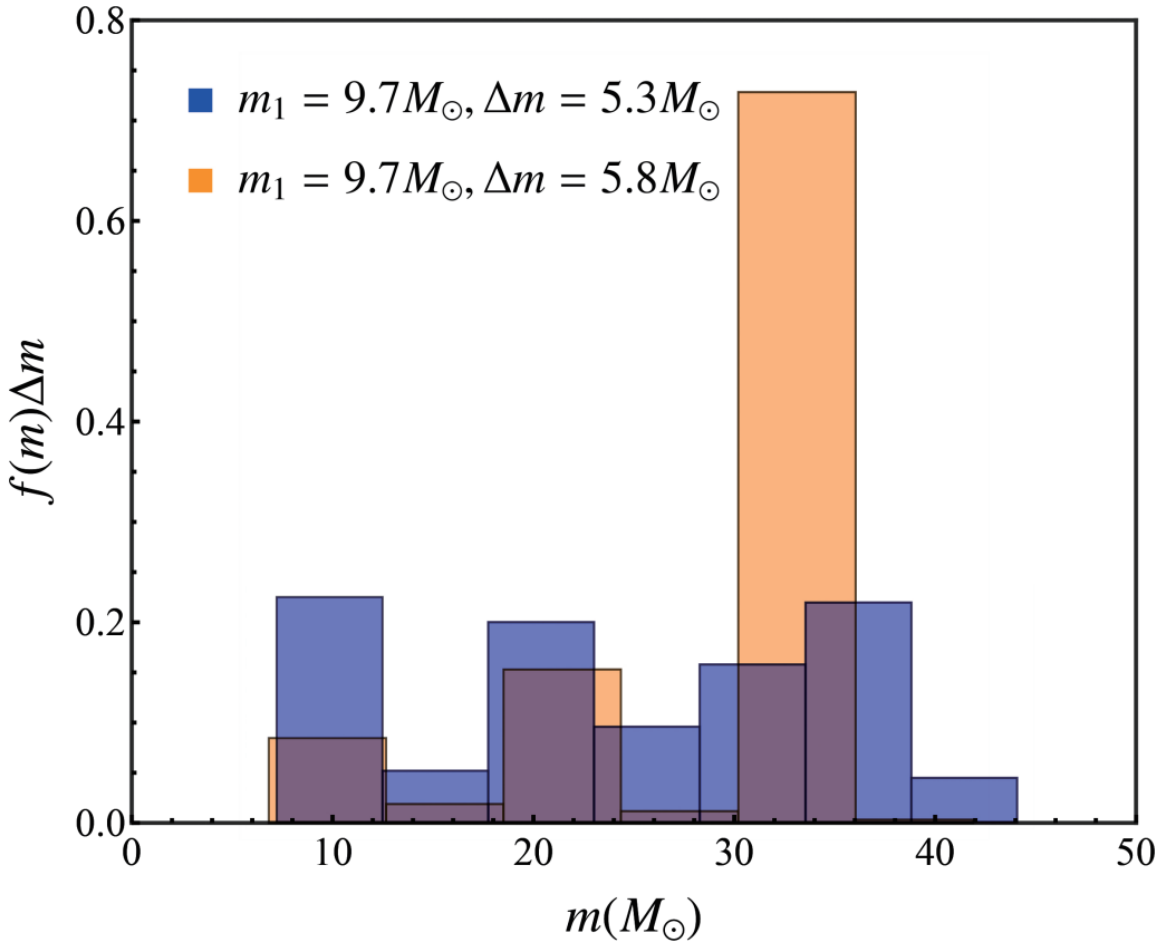


Formation History

Redshift Environment SGWB

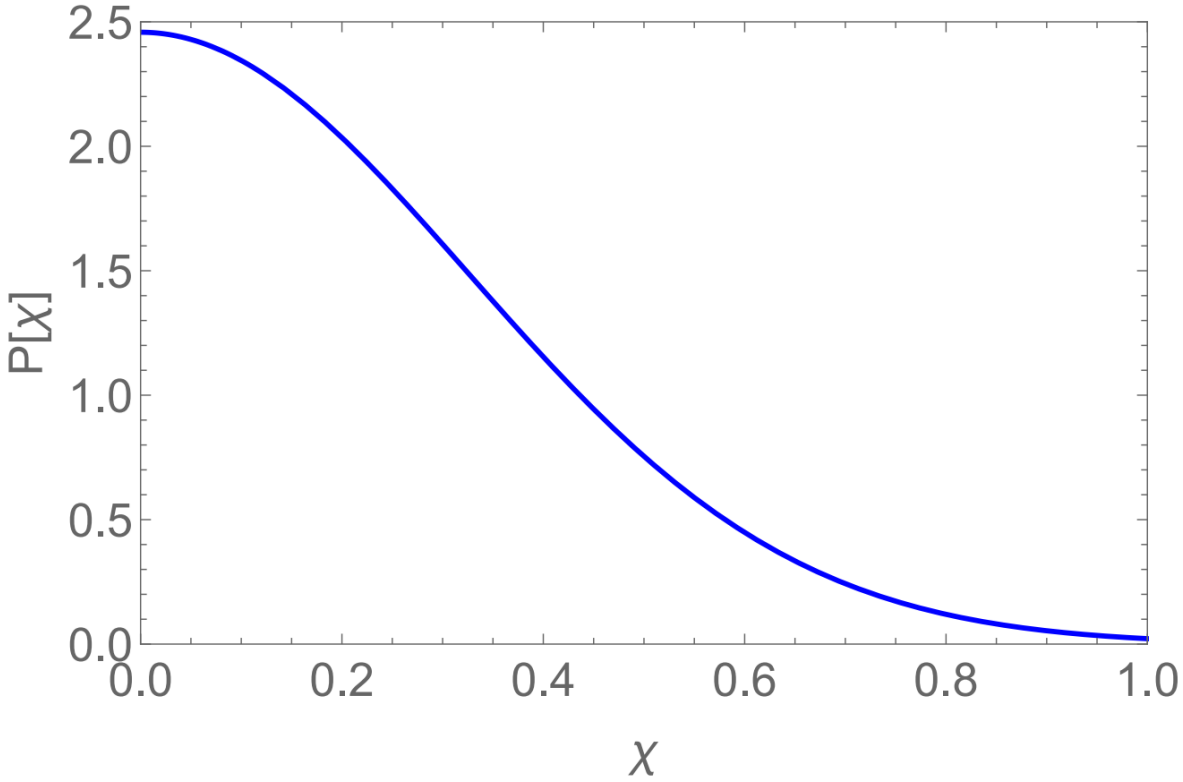
Mass Distribution



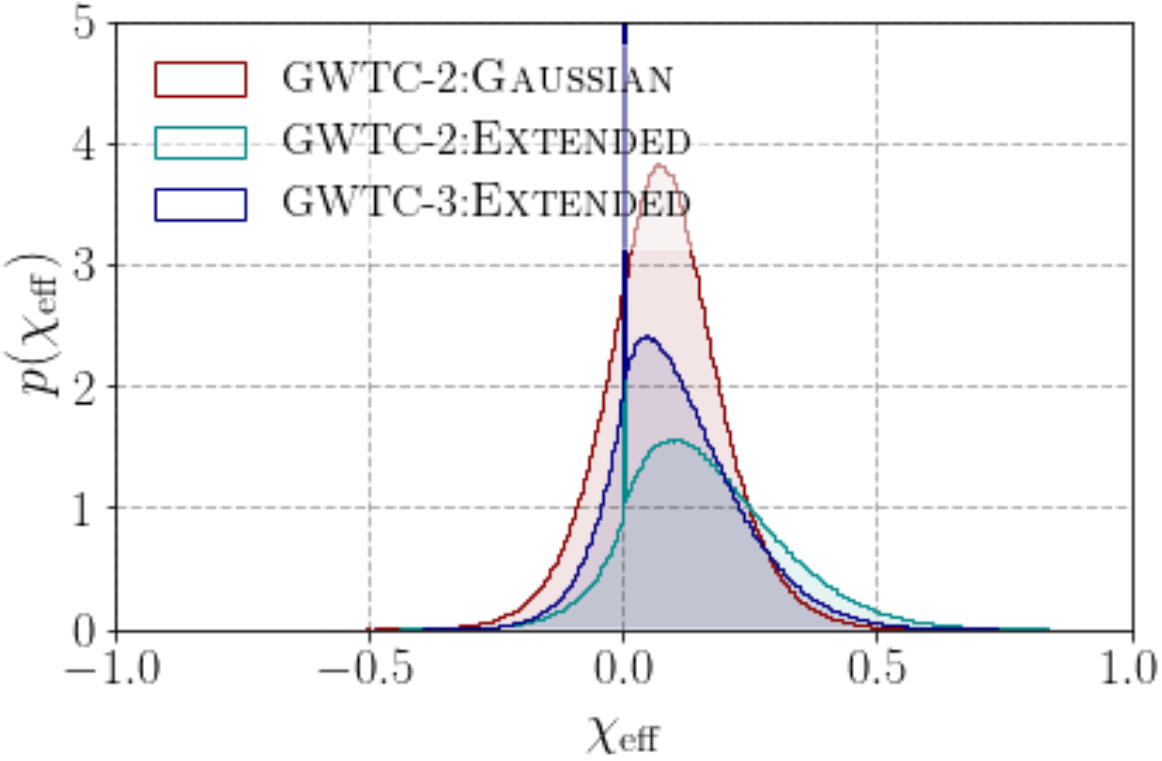


Wang, X., Zhang, Y.L., Kimura, R. and Yamaguchi, M. (2023)

Spin Distribution

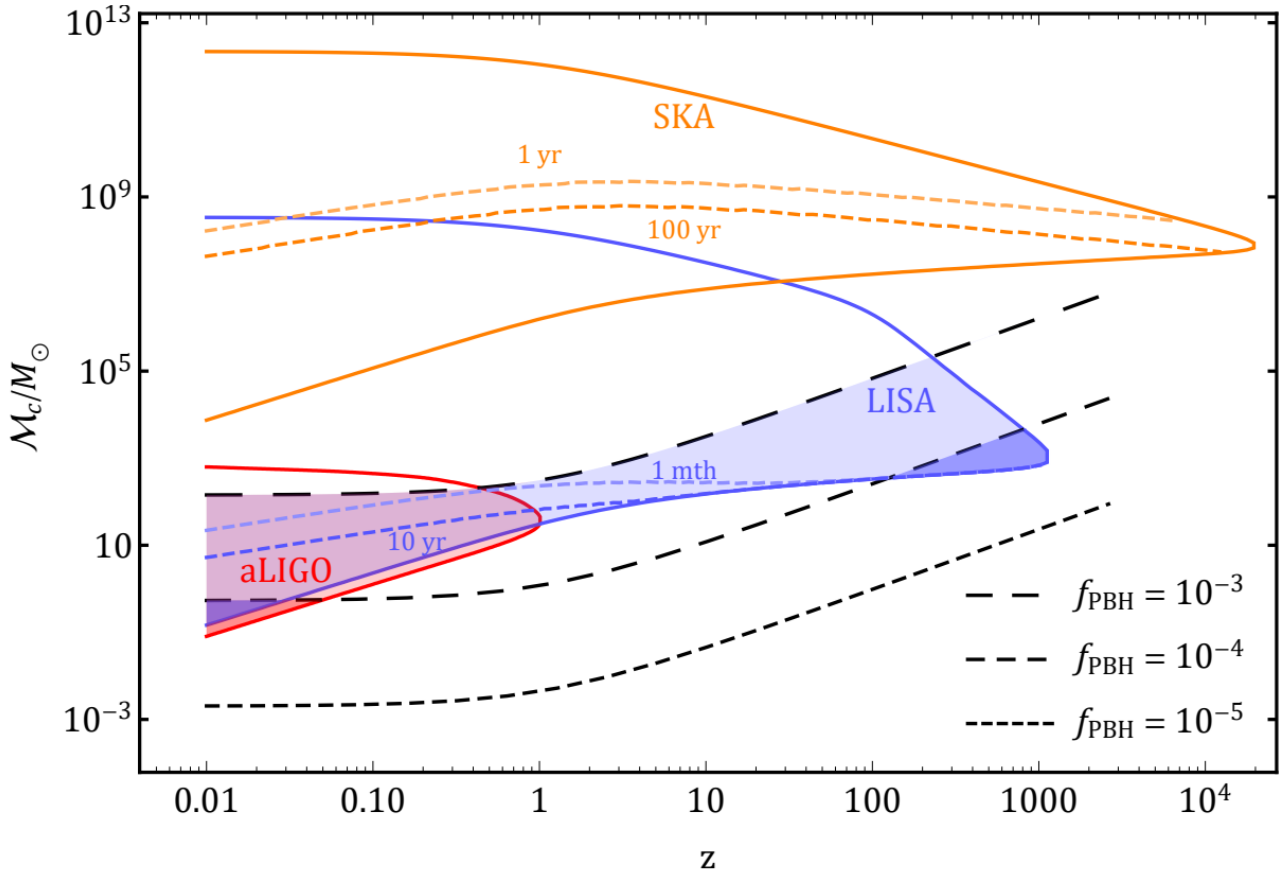


Sasaki, M., Suyama, T., Tanaka, T. and Yokoyama, S. (2018)



Tong, H., Galaudage, S. and Thrane, E. (2022)

Redshift



Ding, Q. (2021)

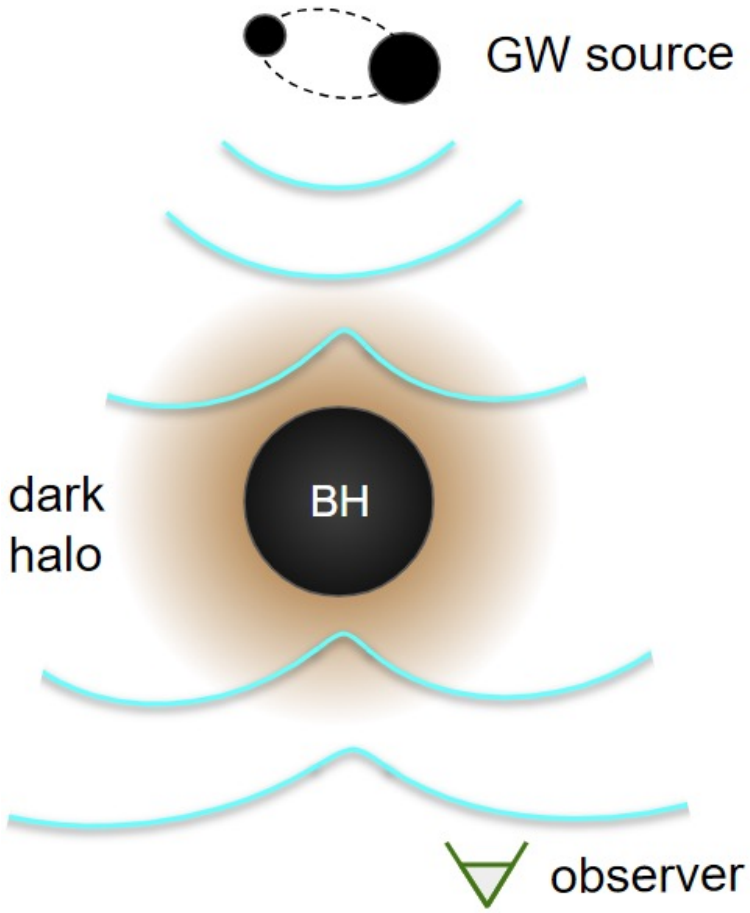
Astrophysical BH

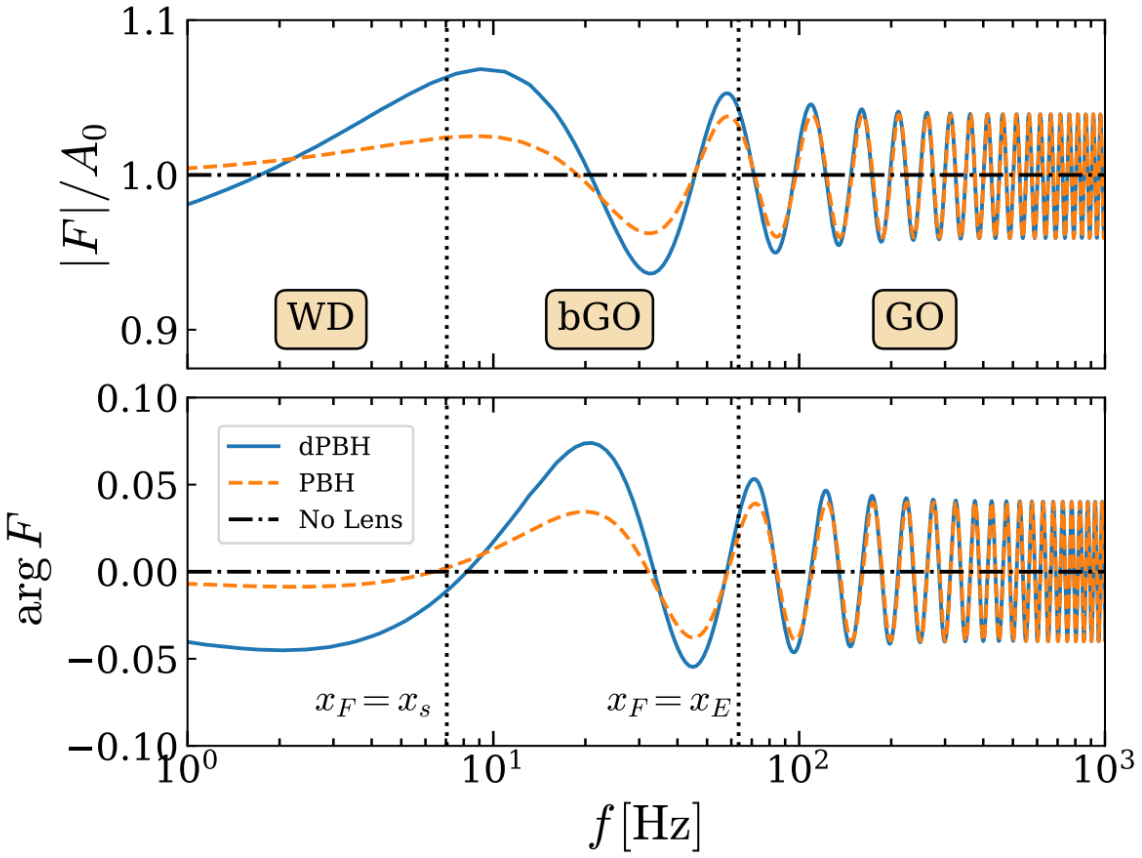
$z < \mathcal{O}(10)$

Primordial Black Hole

$z < z_f$

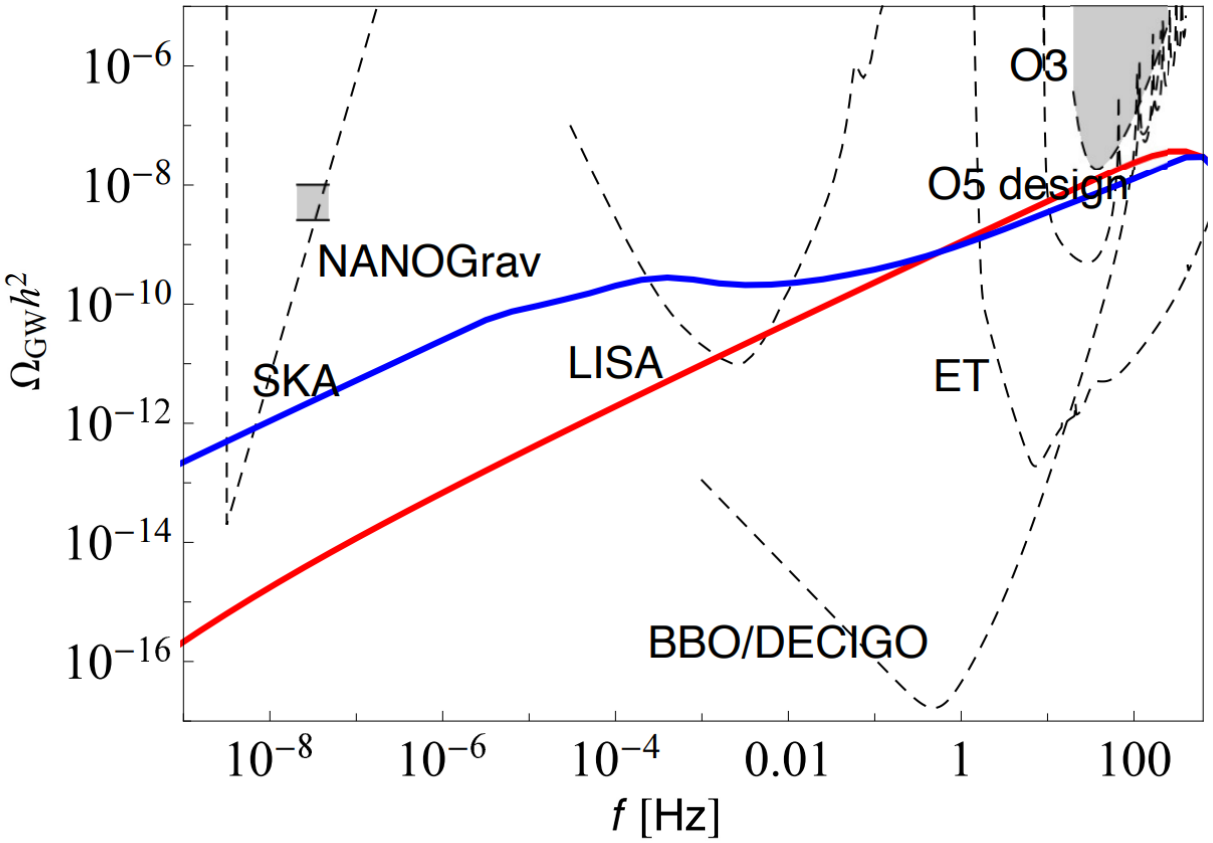
Environment



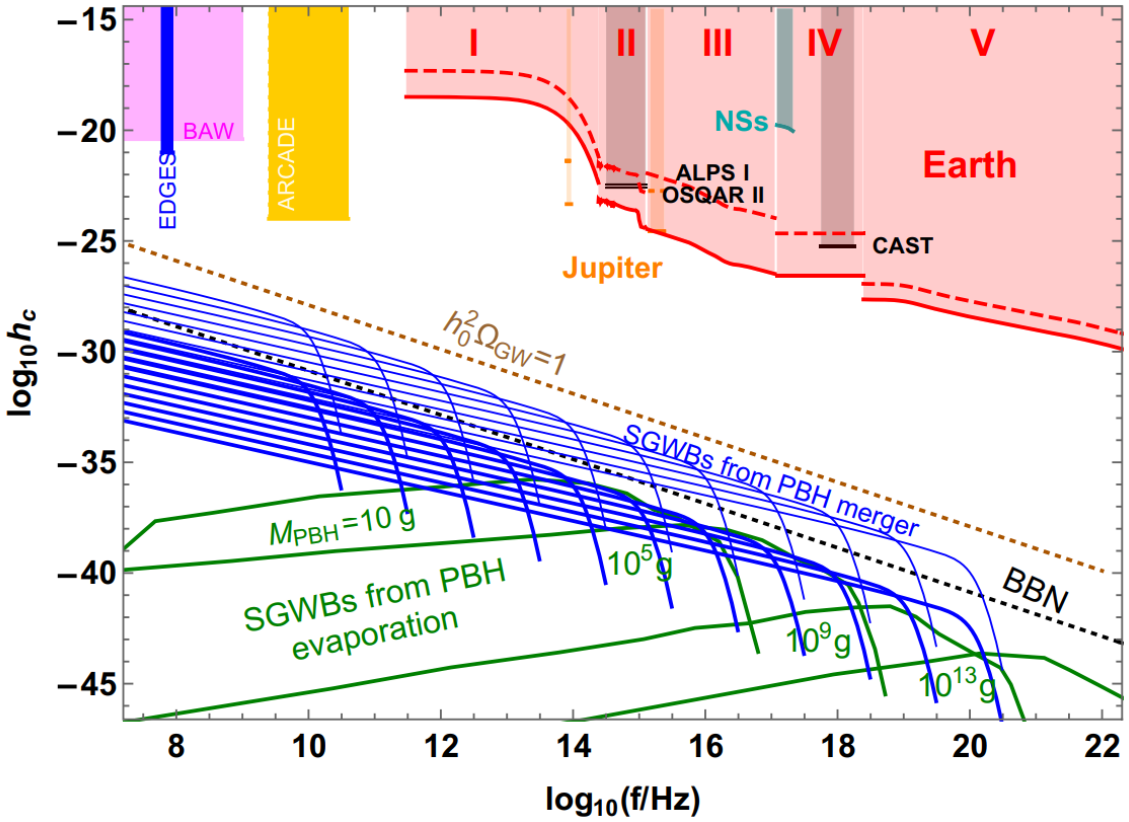


Choi, H.G., Jung, S., Lu, P. and Takhistov, V. (2024)

Stochastic Gravitational Wave Background



LISA Cosmology Working Group. (2025)



Liu, T., Ren, J. and Zhang, C. (2024)

What’s next?

Step into Gravitational Wave Cosmology!

Image credit: Carl Knox, OzGrav, SUT & SARAO

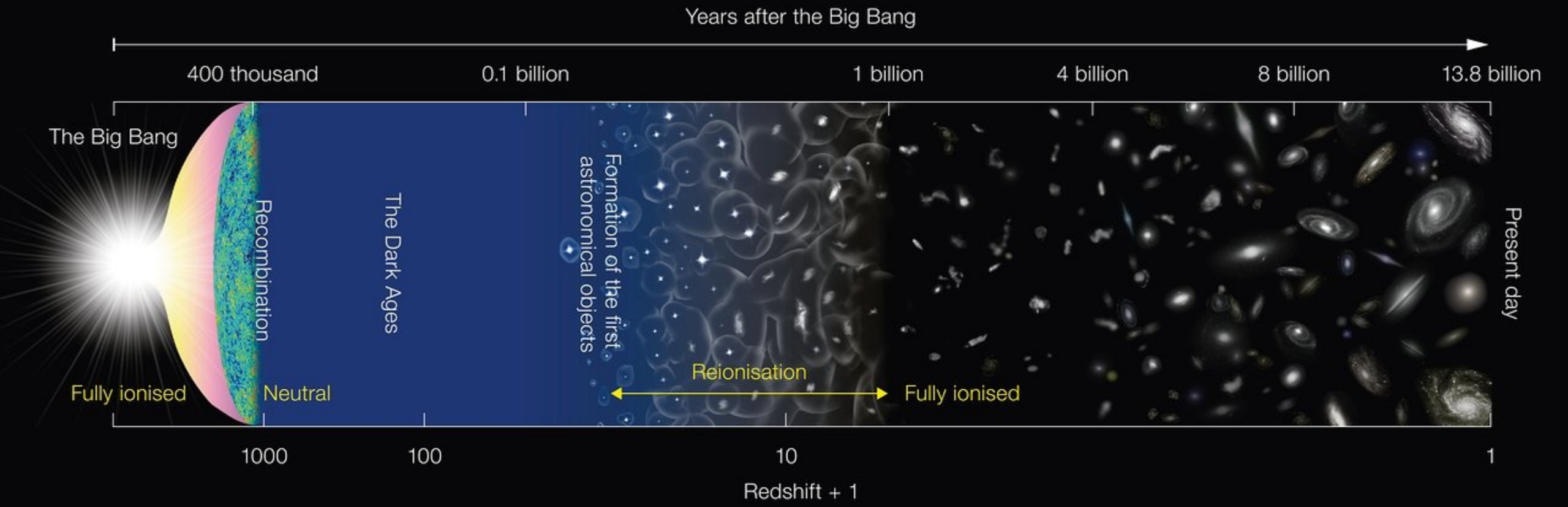
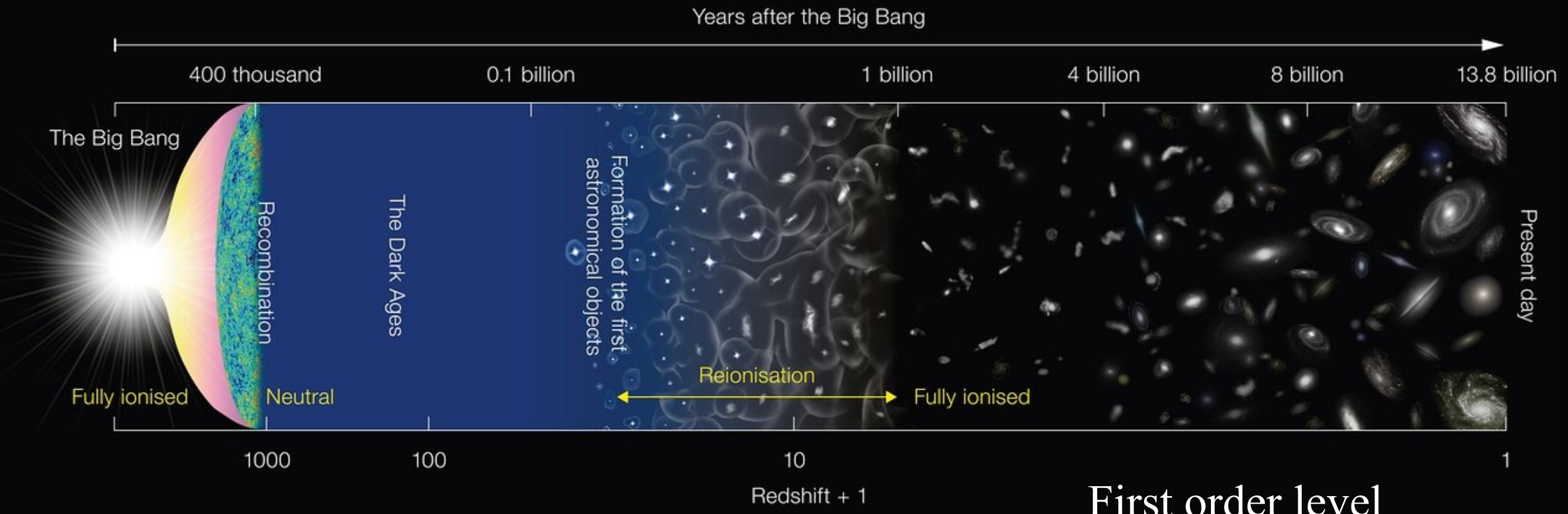


Image Credit: NAOJ

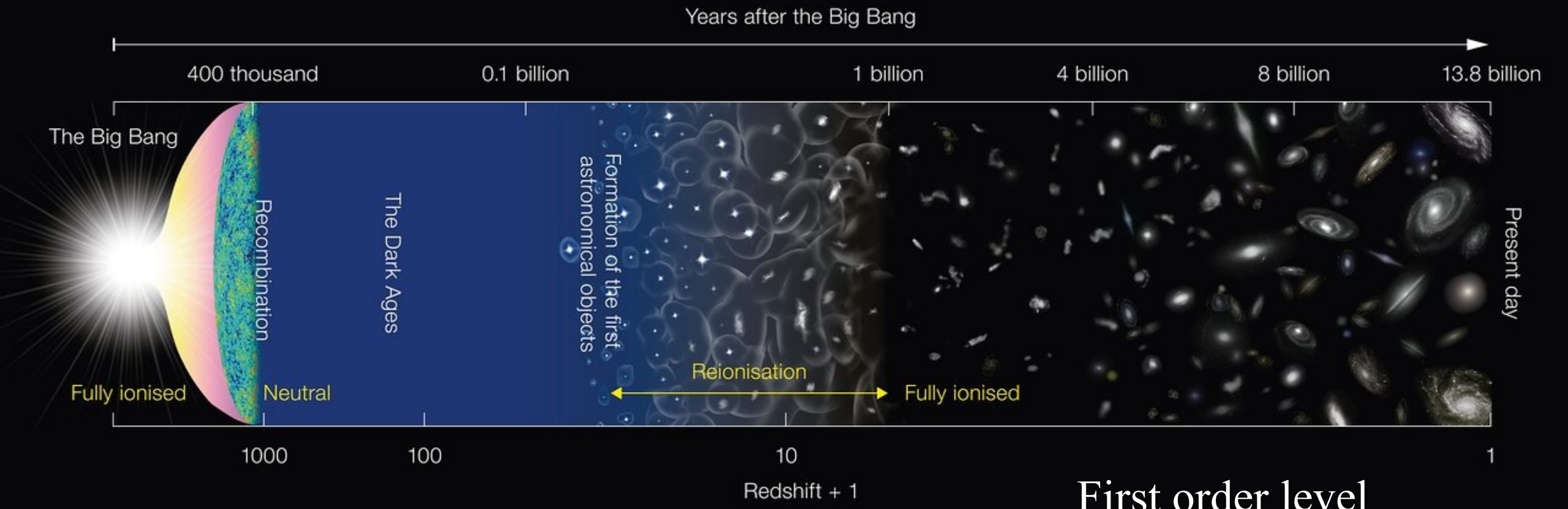
Background level
Hubble expansion rate



First order level
Structure formation

Image Credit: NAOJ

Background level
Hubble expansion rate

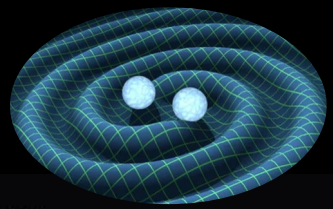


First order level
Structure formation

Ding, Q. (2024)

Image Credit: NAOJ

73.04 ± 1.04 km/s/Mpc



Years after the Big Bang

400 thousand | 0.1 billion | 1 billion | 4 billion | 8 billion | 13.8 billion

The Big Bang

Recombination

The Dark Ages

Formation of the first astronomical objects

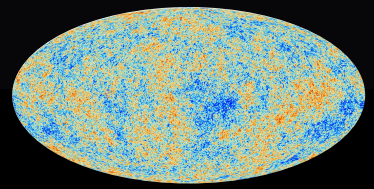
Reionisation

Present day

Fully ionised | Neutral | Fully ionised

1000 | 100 | 10 | 1

Redshift + 1



67.4 ± 0.5 km/s/Mpc

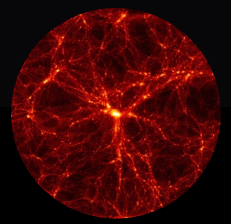


Image Credit: NAOJ

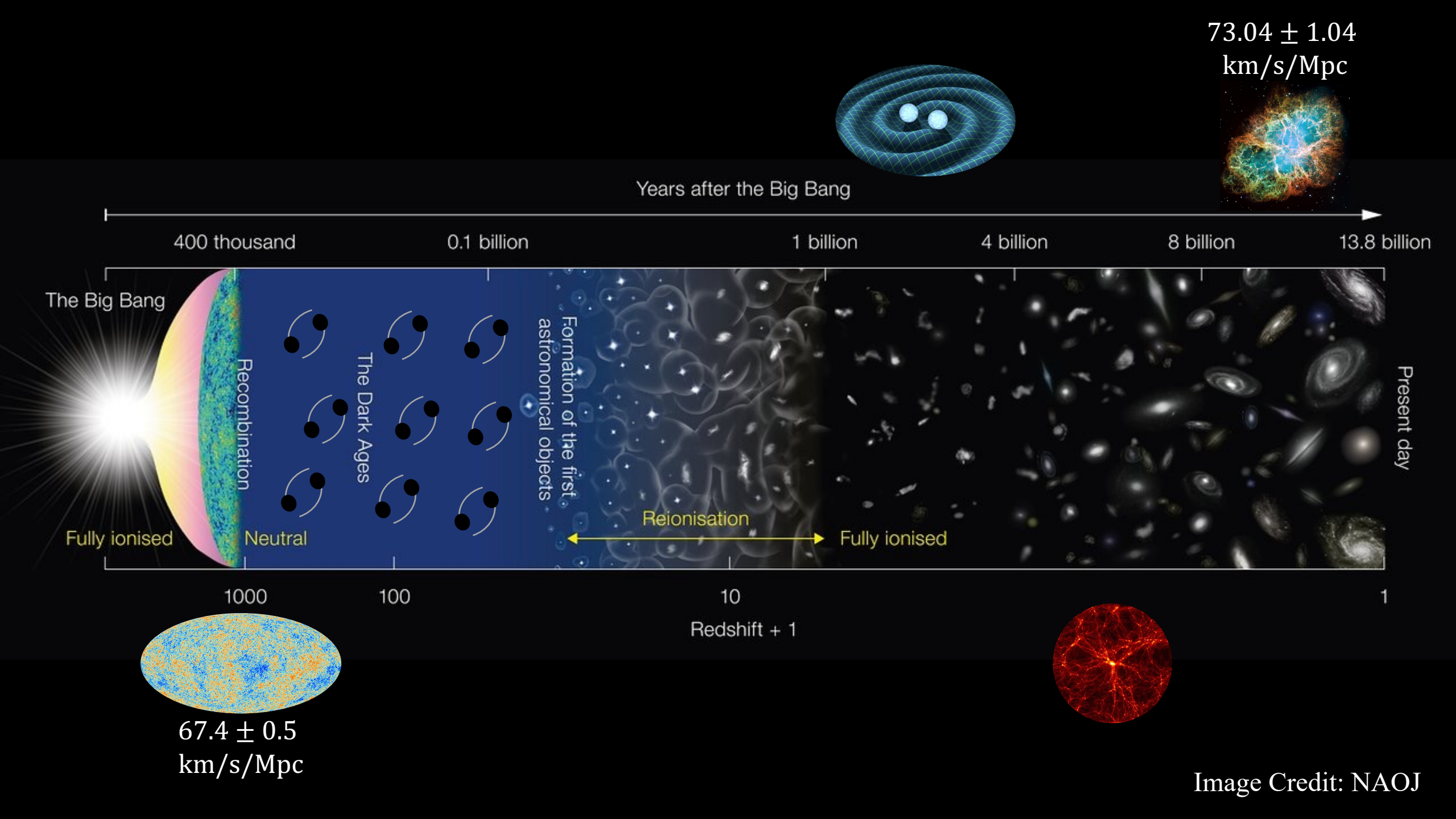
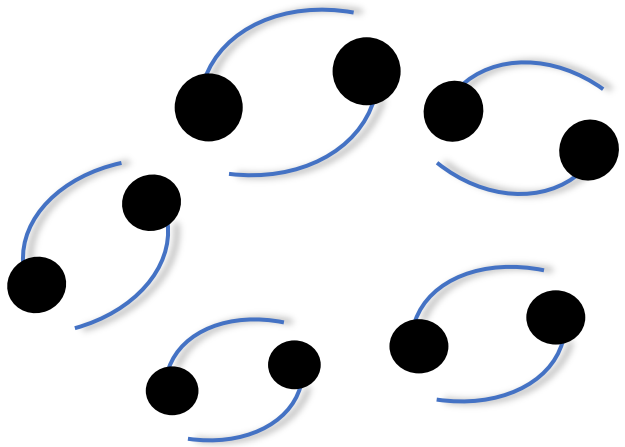

73.04 ± 1.04
km/s/Mpc
Years after the Big Bang
400 thousand
0.1 billion
1 billion
4 billion
8 billion
13.8 billion
The Big Bang
Recombination
The Dark Ages
Formation of the first astronomical objects
Reionisation
Present day
Fully ionised
Neutral
Fully ionised
1000
100
10
1
Redshift + 1
67.4 ± 0.5
km/s/Mpc
Image Credit: NAOJ

Image Credit: ESO

$$h(t) = \frac{4}{d_L(z)}\left(\frac{G\mathcal{M}_z}{c^2}\right)^{5/3}\left(\frac{\pi f(t)}{c}\right)^{2/3}\cos\Phi(t)$$

$$d_L = \frac{1+z}{H_0}\int_0^z \frac{c}{E(z')}dz' \qquad \mathcal{M}_z = \mathcal{M}(1+z)$$

Image Credit: ESO



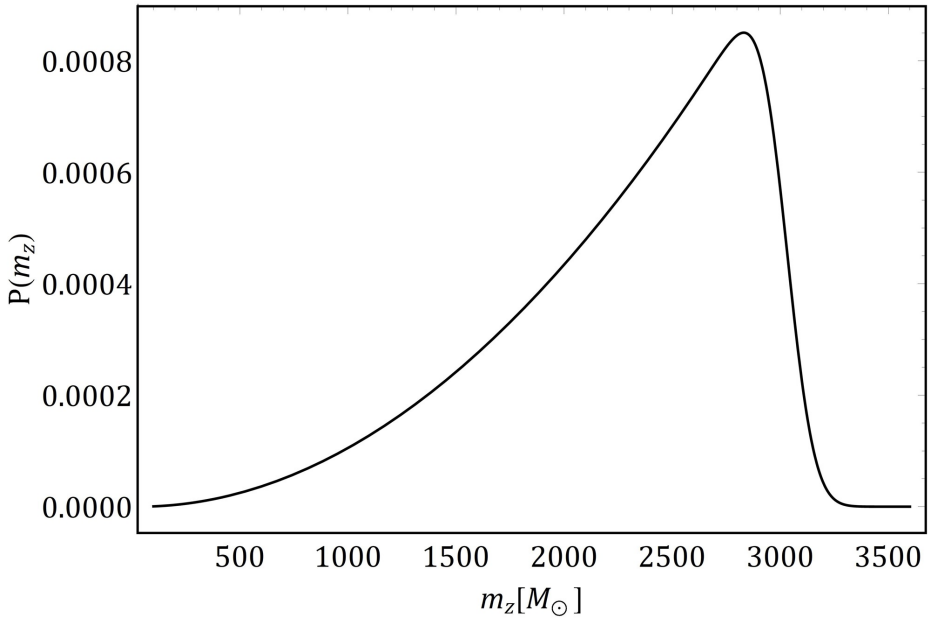
(m_z^i, d_L^i)

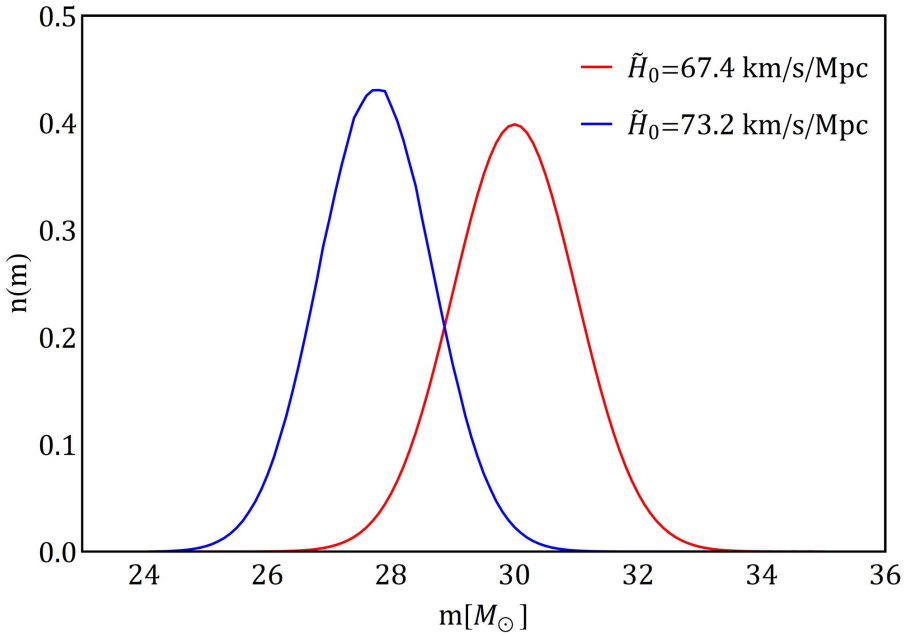
$$d_L^i = \frac{1+z_i}{H_0}\int_0^{z_i}\frac{c}{E(z')}dz'$$

Assume a Hubble parameter $\tilde{H}_0$

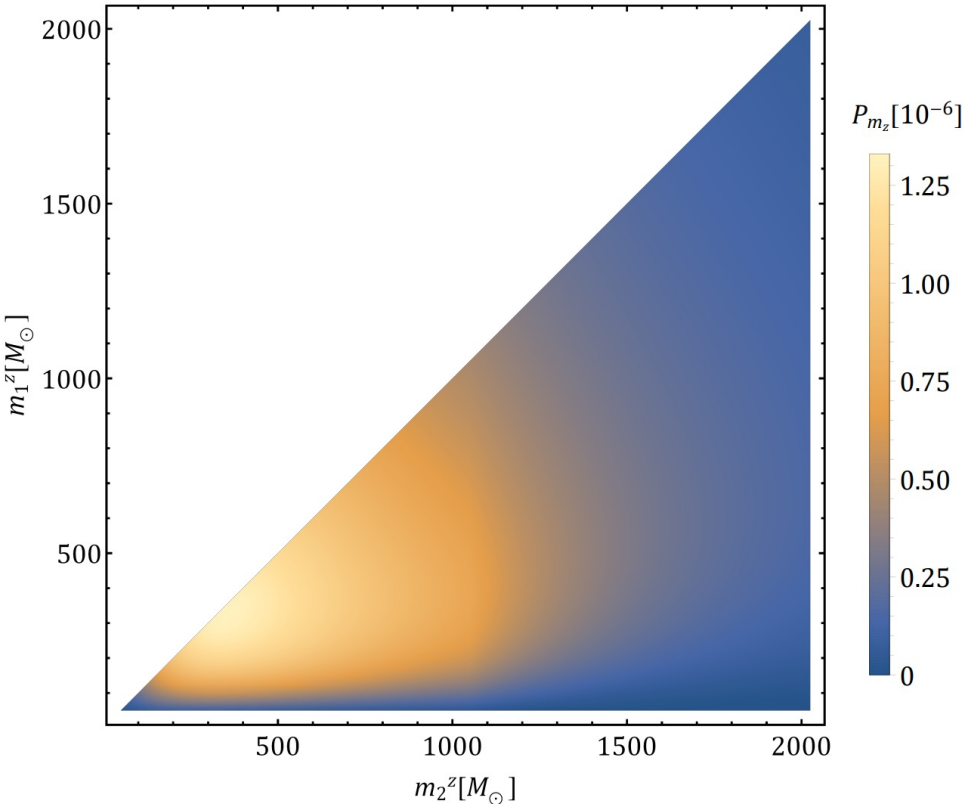
$$z_i = z(d_L^i; \tilde{H}_0)$$

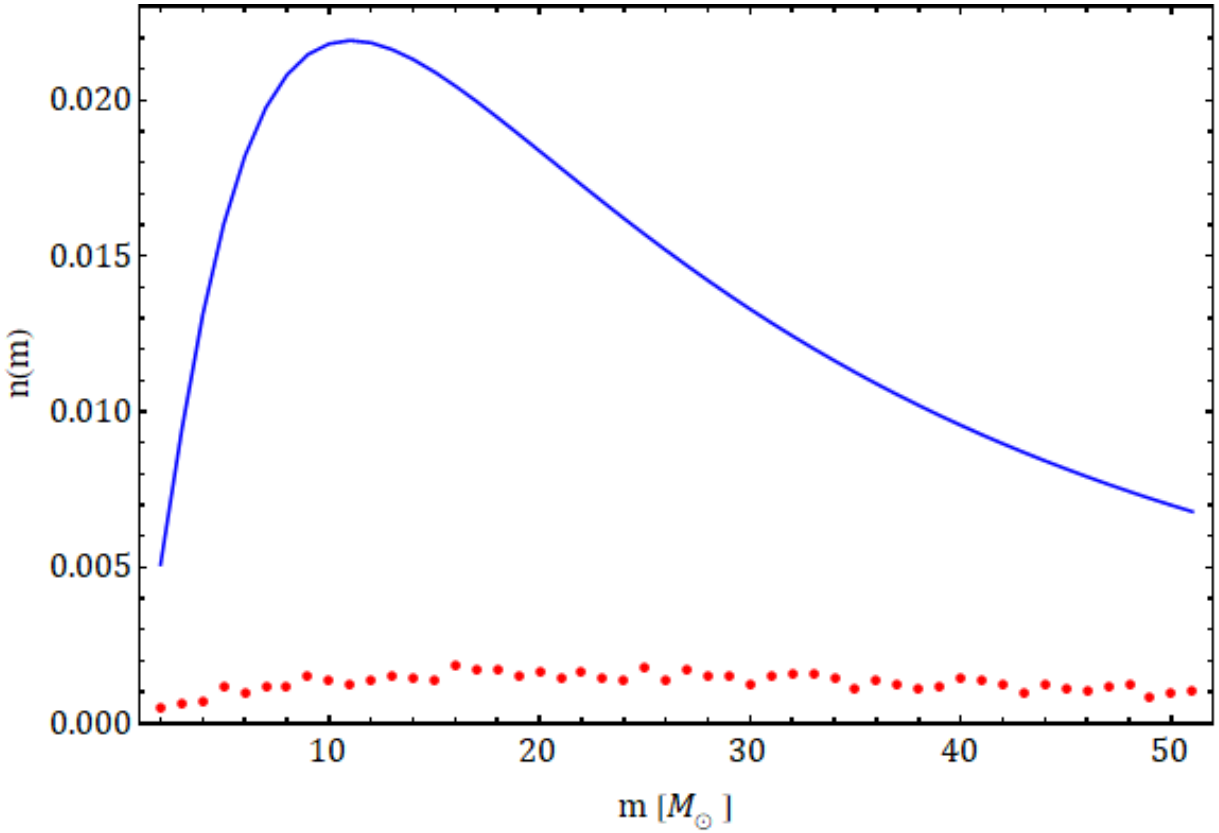
$$m^i = m_z^i/(1+z_i)$$





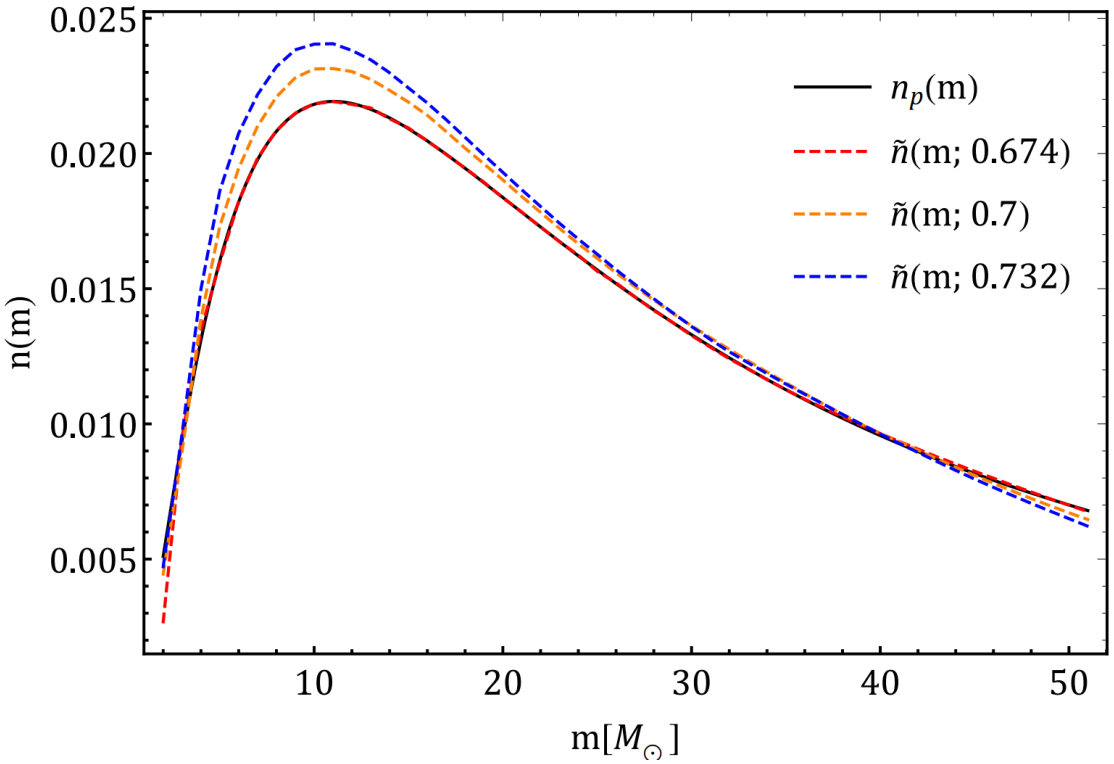
Reconstruct PBH Mass Function

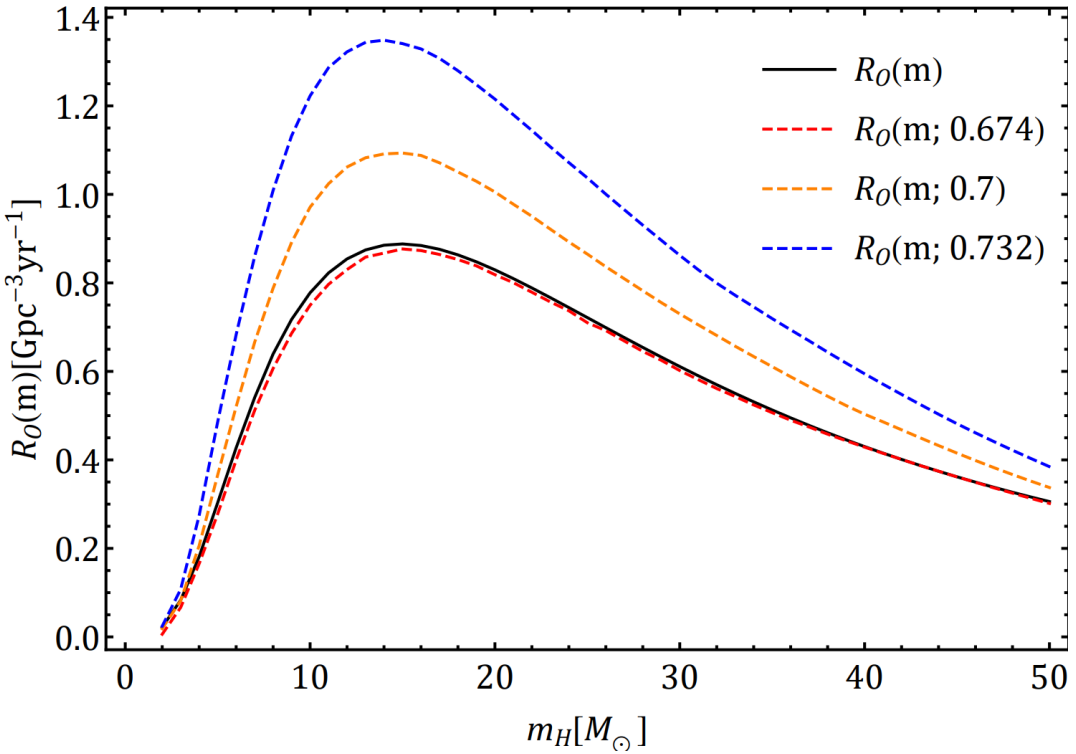




$$P_O(m_1^z, m_2^z) = \int_0^\infty n_p\left(\frac{m_1^z}{1+z}\right) n_p\left(\frac{m_2^z}{1+z}\right) W\left(\frac{m_1^z}{1+z}, \frac{m_2^z}{1+z}; z\right) \frac{p(z;\tilde{H}_0)}{(1+z)^2} dz$$

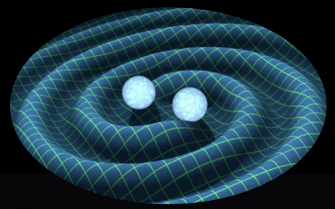
PBH Mergers as a Probe of Hubble Parameter



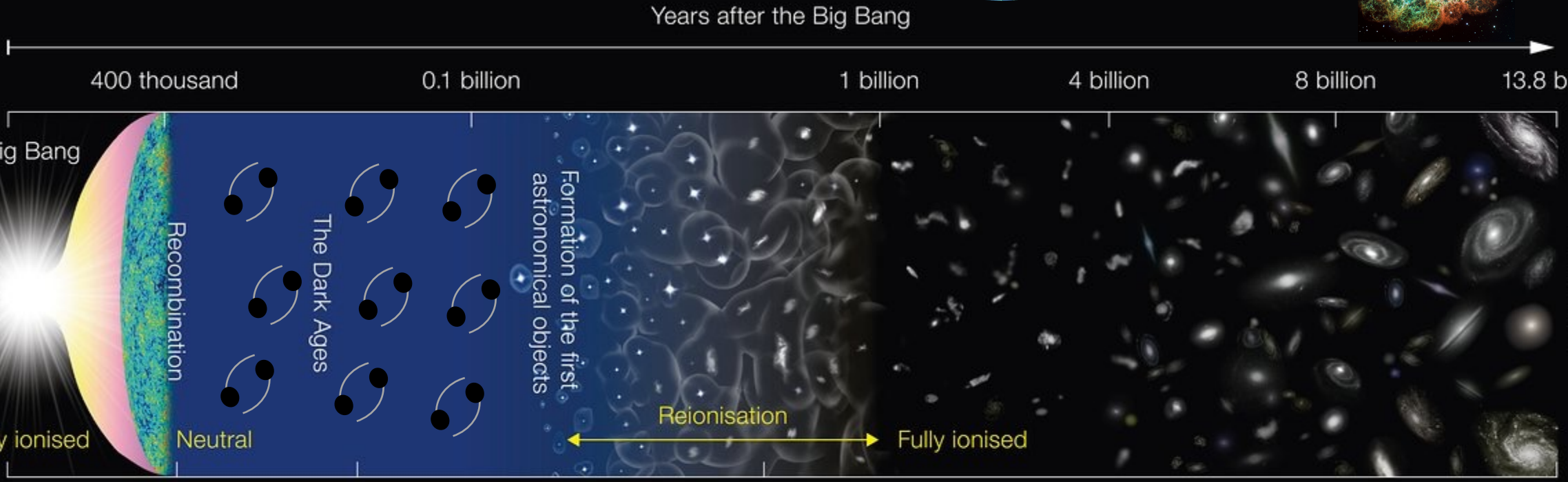


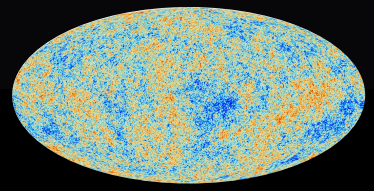
$$R_{ij} = n_{\mathrm{PBH}} \min\left(n(m_i; \tilde{H}_0), n(m_j; \tilde{H}_0)\right) \Delta_m \frac{dP}{dt}$$

Background level Hubble expansion rate



73.04 ± 1.04 km/s/Mpc





67.4 ± 0.5 km/s/Mpc

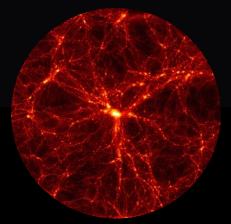
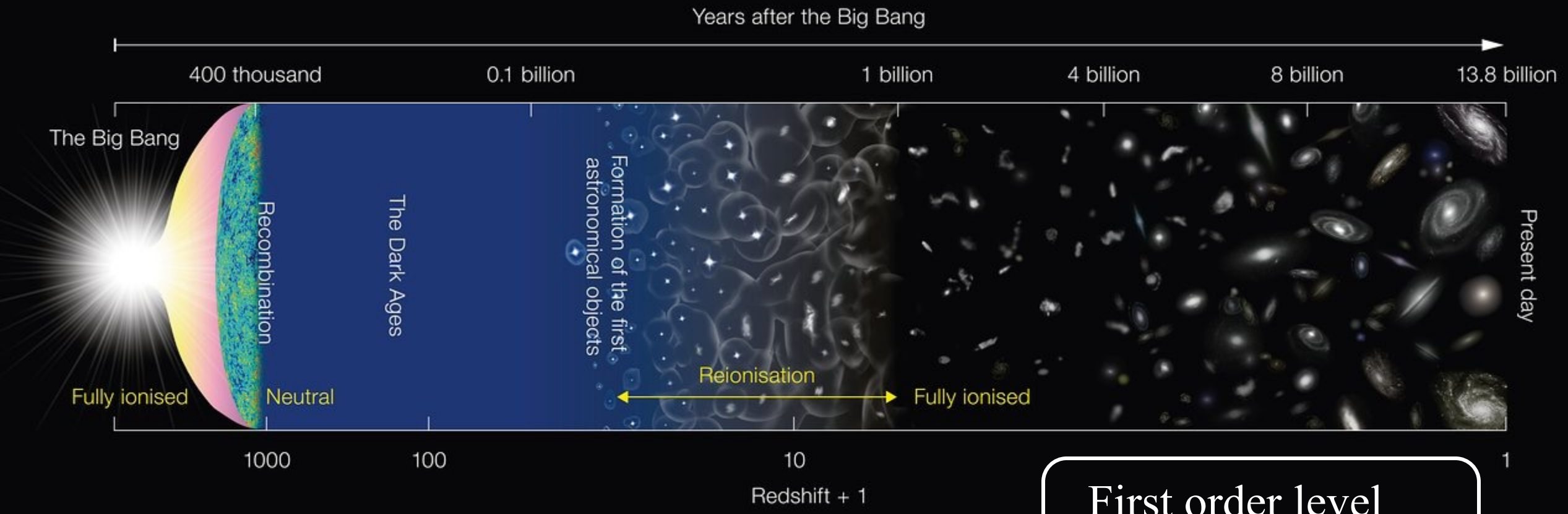


Image Credit: NAOJ

Background level
Hubble expansion rate

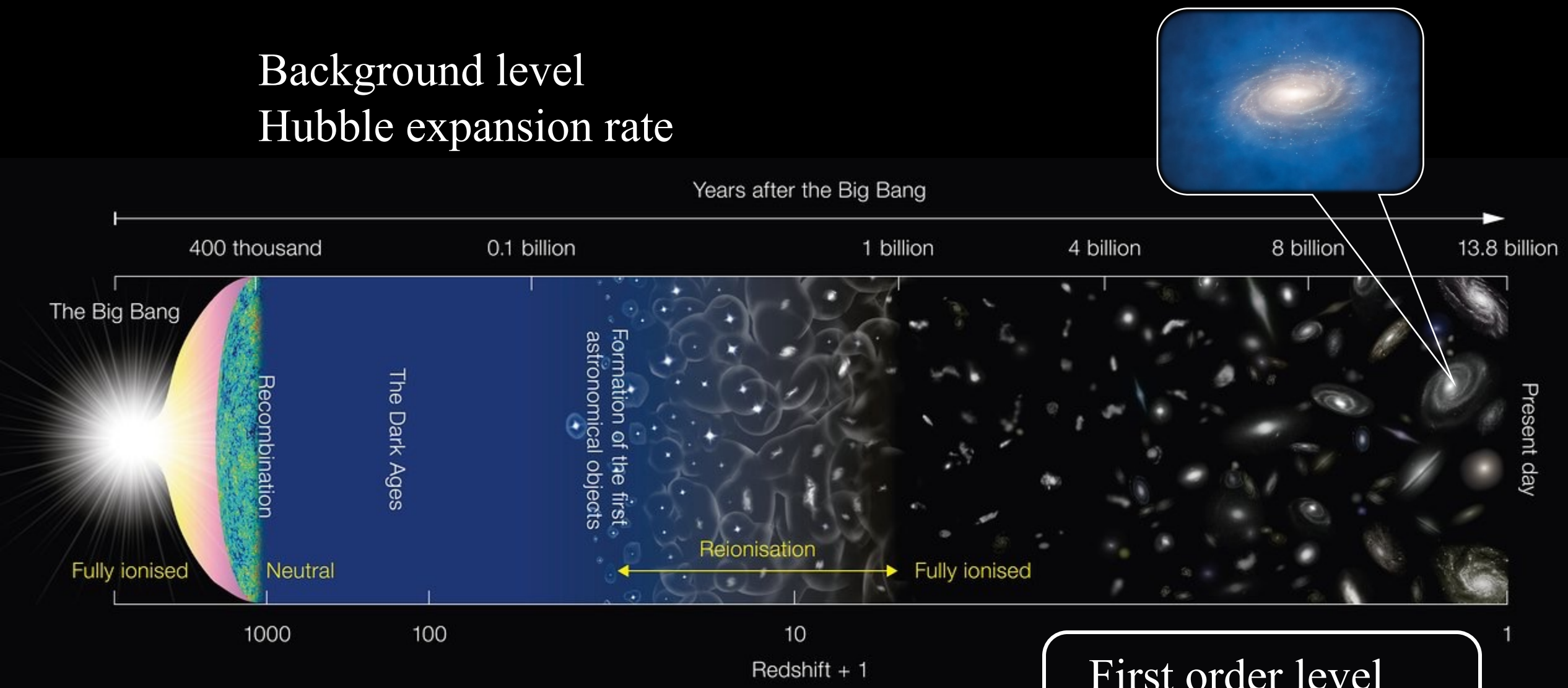


First order level
Structure formation

Ding, Q., He, M. and Takhistov, V. (2025)

Image Credit: NAOJ

Background level
Hubble expansion rate

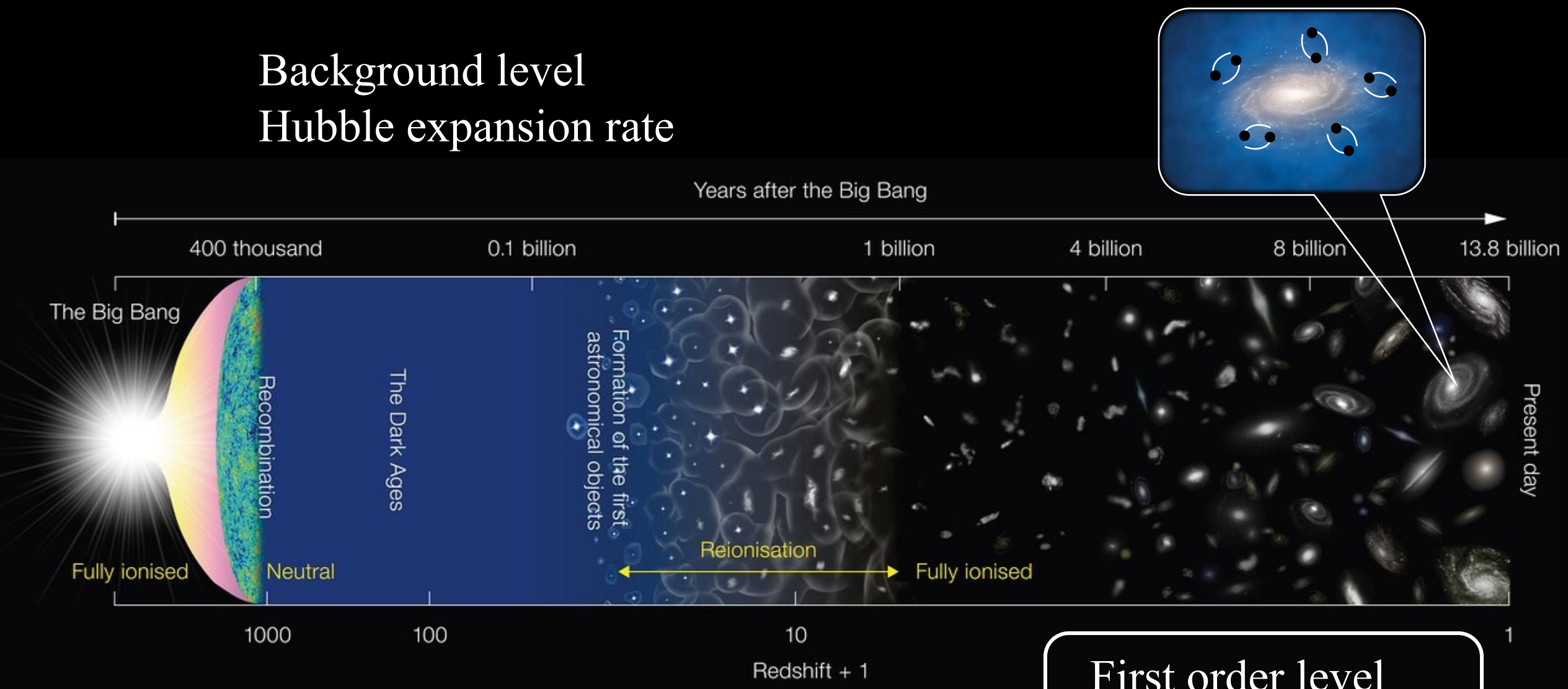


First order level
Structure formation

Ding, Q., He, M. and Takhistov, V. (2025)

Image Credit: NAOJ

Background level
Hubble expansion rate



First order level
Structure formation

Ding, Q., He, M. and Takhistov, V. (2025)

Image Credit: NAOJ

$\rho_{\mathrm{DM}}^{0} = 2.2\times10^{-27}\mathrm{kg/m^3}$

$\rho_{\mathrm{DM}}^{\mathrm{sun}} = 5.9\times10^{-22}\mathrm{kg/m^3}$

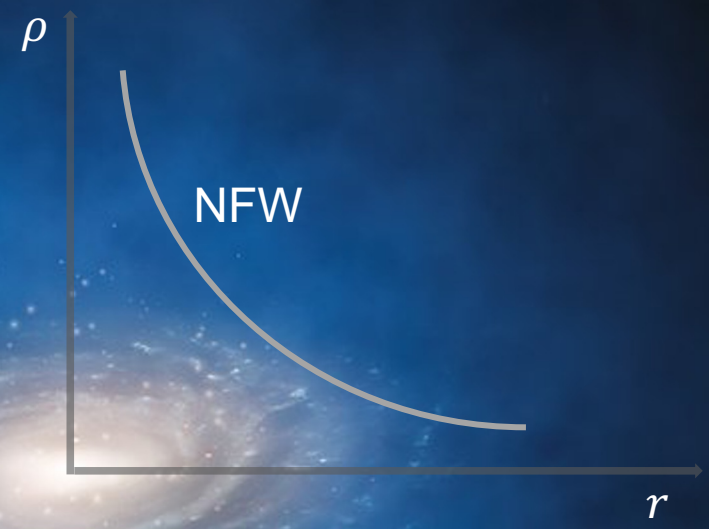


Image Credit: ESO

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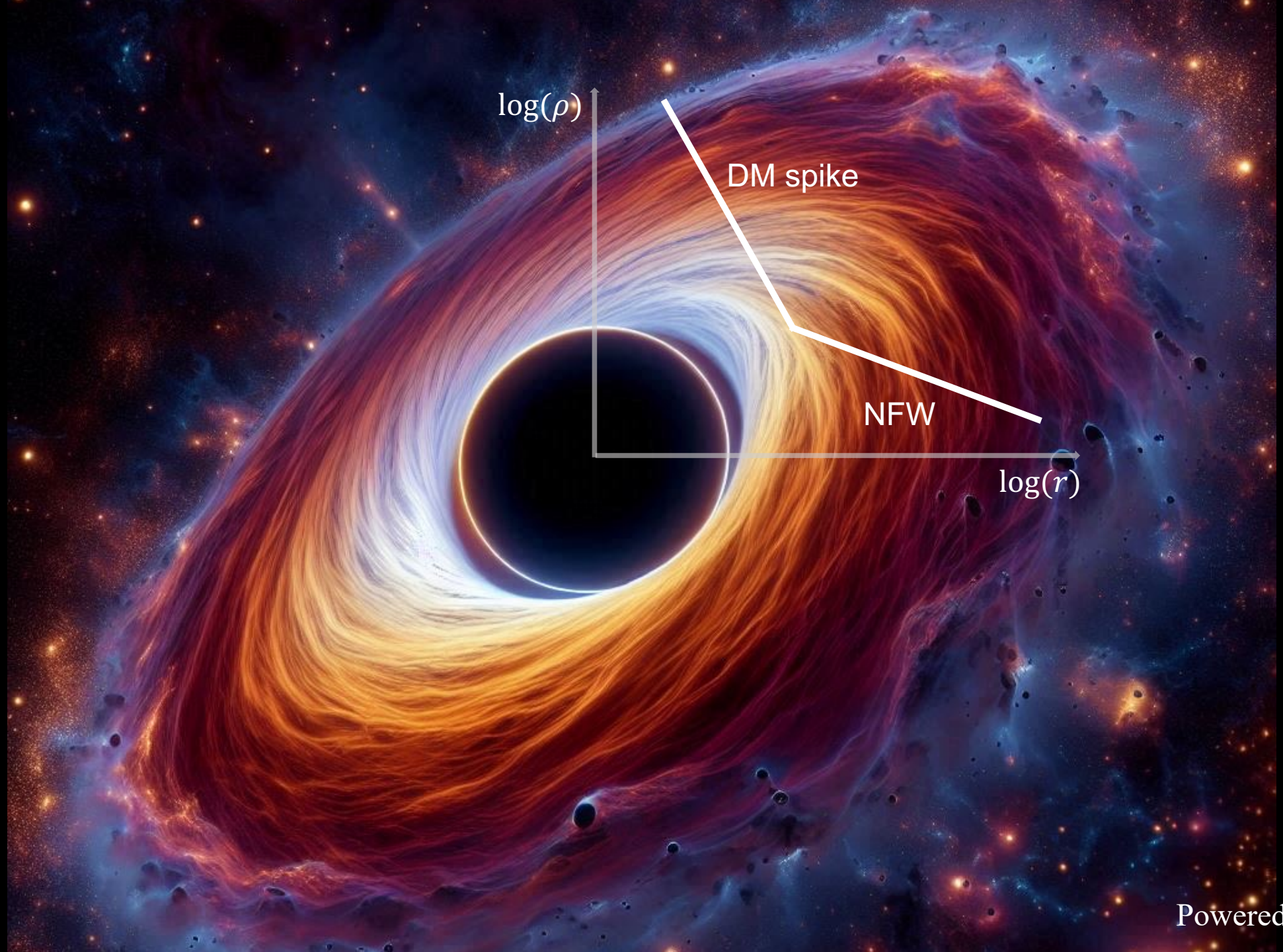
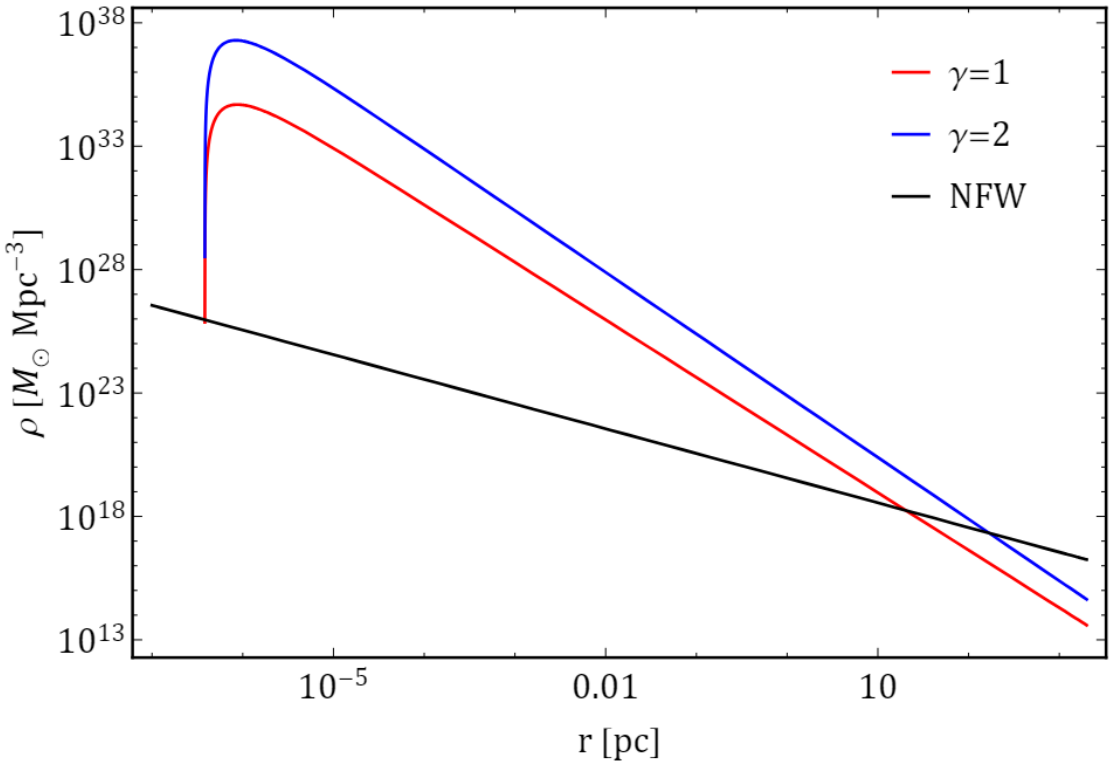
Image Credit: ESO


Powered by DALL·E 3



Powered by DALL·E 3

Dark Matter Spike



Fokker-Planck Equation

$$\frac{\partial[f(E,t)g(E)]}{\partial t} = -\frac{\partial \mathcal{F}(E,t)}{\partial E},$$

$$-\mathcal{F}(E,t) \equiv D_{EE}(E)\frac{\partial f(E,t)}{\partial E} + D_E(E) f(E,t).$$

$$\rho(r) \simeq \rho_0 (r_0/r)^{\gamma}$$

$$\rho(r) = 4\pi \int_{\Phi(r)}^{0} dE\, f(E)\, \sqrt{2[E-\Phi(r)]}$$

$$\rho_{\rm sp}(r) = \rho_R \left(1 - \frac{4R_{\rm s}}{r}\right)^3 \left(\frac{R_{\rm sp}}{r}\right)^{\gamma_{\rm sp}}$$

$$\gamma_{\rm sp} = \frac{9-2\gamma}{4-\gamma}$$

PBH Merger Rate in DM Spike

$$\Gamma = \int_{M_{\rm min}}^{M_{\rm max}} N_{\rm sp}(M_{\rm SMBH}) \frac{dn}{dM_{\rm SMBH}} dM_{\rm SMBH}$$

PBH merger rate in one spike

$$N_{\rm sp} = \int_{4R_s}^{R_{\rm sp}} \frac{1}{2} \left(\frac{f_{\rm PBH}\rho_{\rm sp}(r)}{M_{\rm PBH}} \right)^2 \sigma_m(r) v_{\rm rel}(r) d^3r$$

$$\sigma_m(r) = 1.4 \times 10^{-14} \left(\frac{M_{\rm PBH}}{30\,M_\odot} \right)^2 \left(\frac{v_{\rm rel}(r)}{200\,{\rm km\,s^{-1}}} \right)^{-\frac{18}{7}} {\rm pc}^2$$

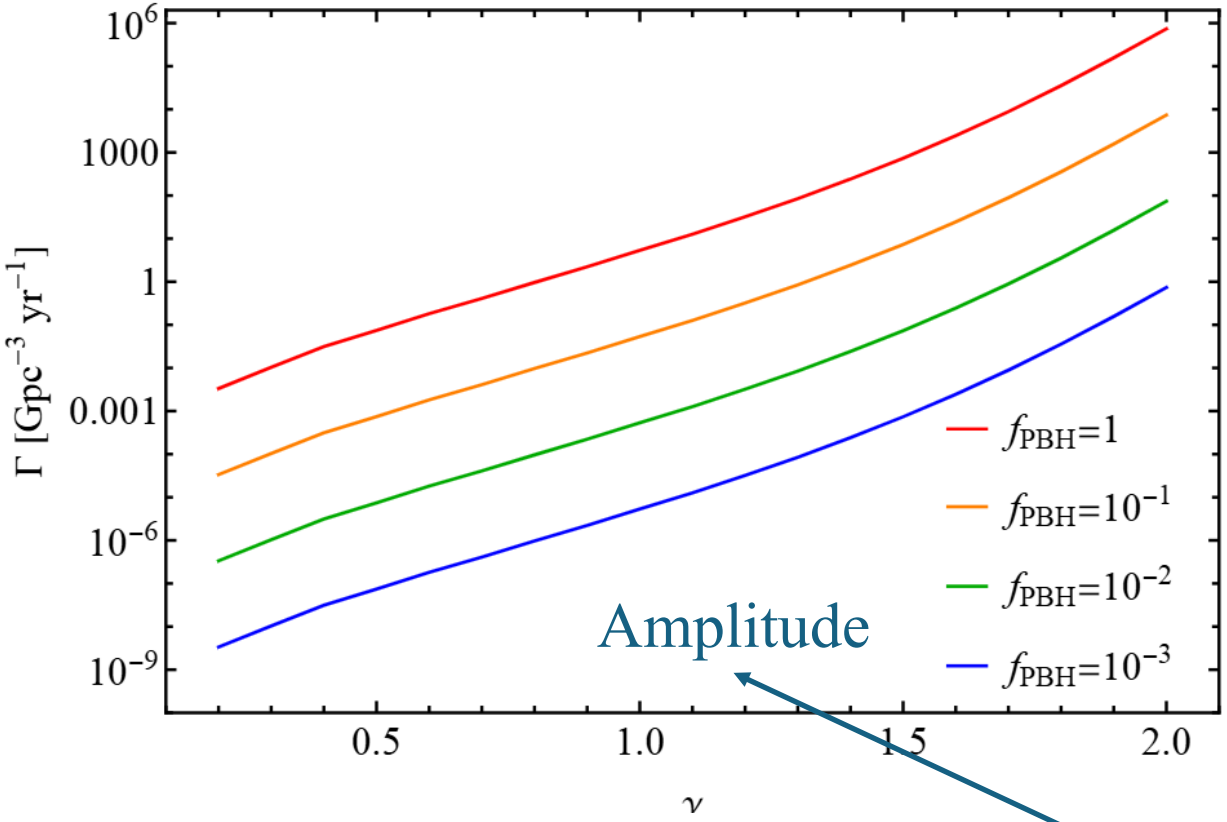
SMBH number density distribution

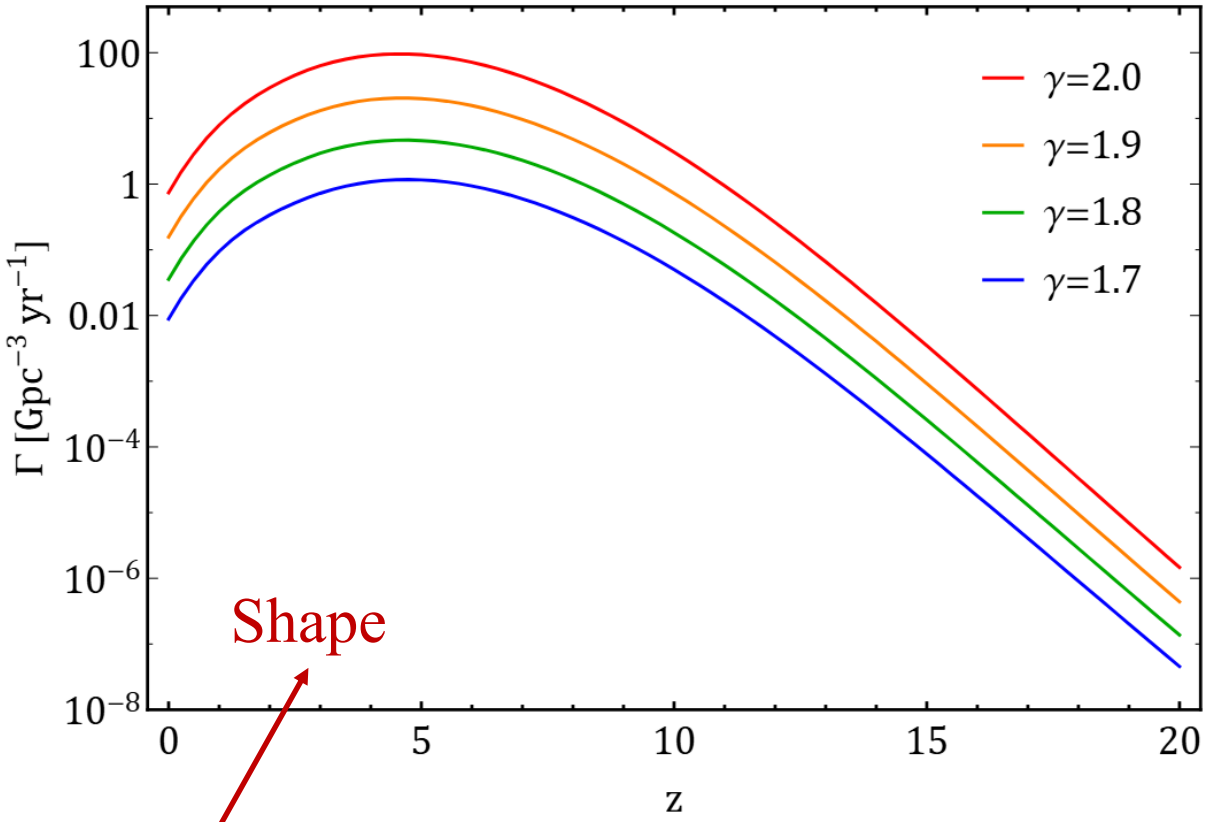
$$\frac{dn}{dM_{\rm SMBH}} = \frac{dn}{dM_{\rm vir}} \frac{dM_{\rm vir}}{dM_{\rm SMBH}}$$

$$= f(\sigma) \frac{\rho_m}{M_{\rm vir}} \frac{d\log(\sigma^{-1})}{dM_{\rm vir}} \frac{dM_{\rm vir}}{dM_{\rm SMBH}}$$

$$\log_{10}\left(\frac{M_{\rm SMBH}}{M_\odot} \right) = a + b \log_{10}\left(\frac{\sigma}{200\,{\rm km\,s^{-1}}} \right) - \xi \log_{10}(1+z)\ ,$$

PBH Merger Rate in DM Spike



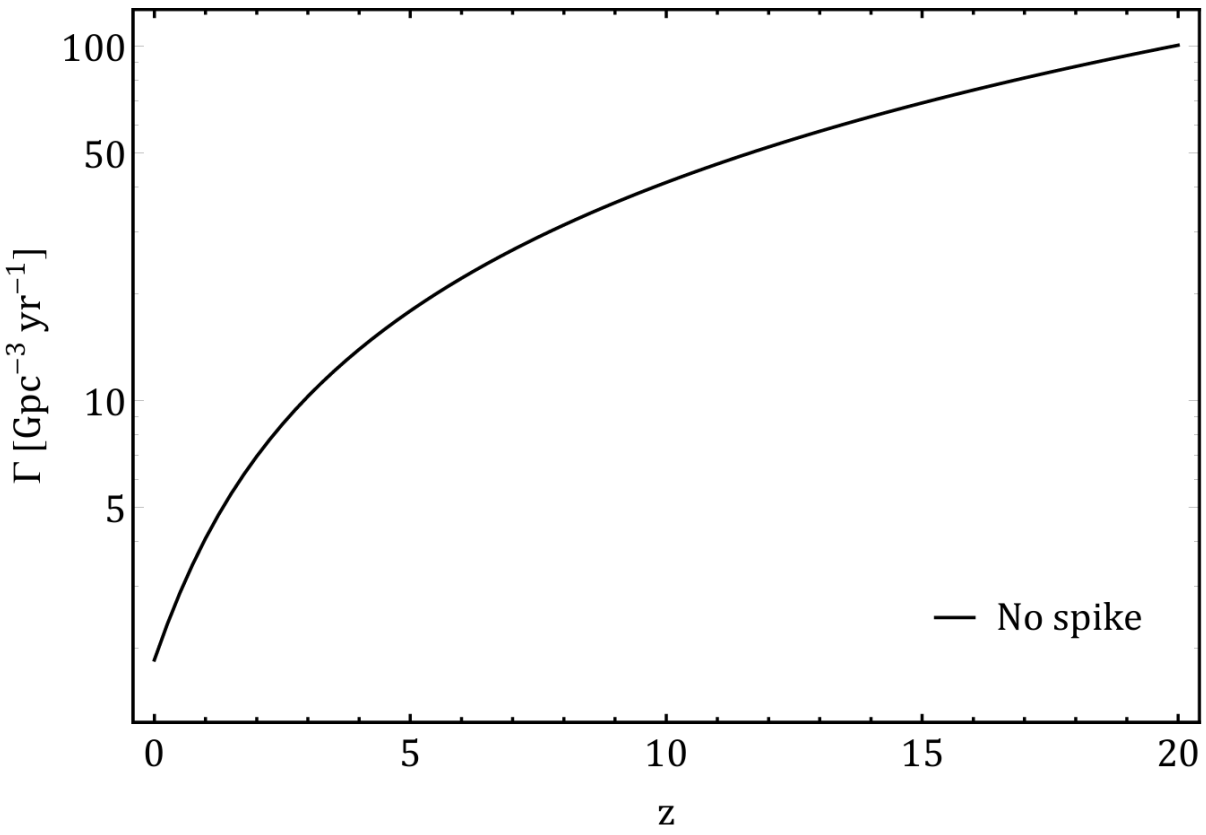


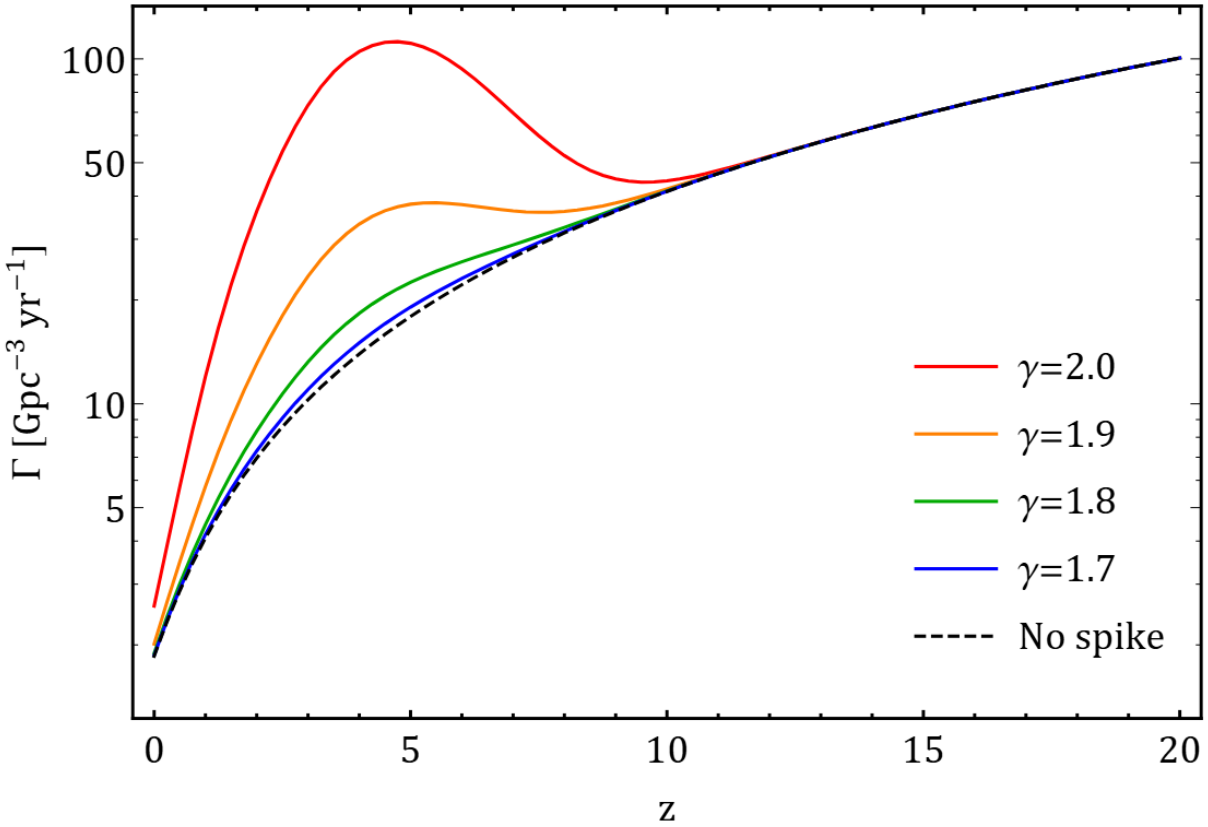
Amplitude

Shape

$$\Gamma = \int_{M_{\rm min}}^{M_{\rm max}} N_{\rm sp}(M_{\rm SMBH}) \frac{dn}{dM_{\rm SMBH}} dM_{\rm SMBH}$$

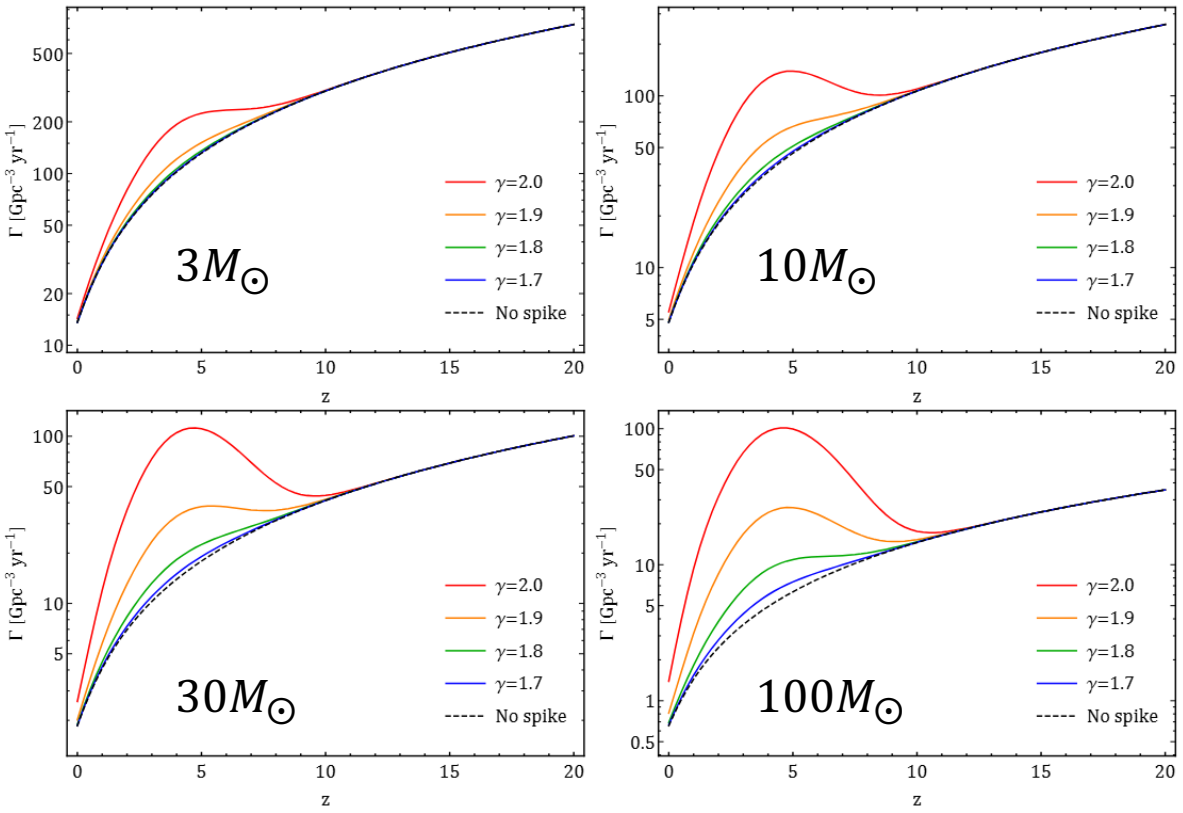
PBH Merger Rate Evolution

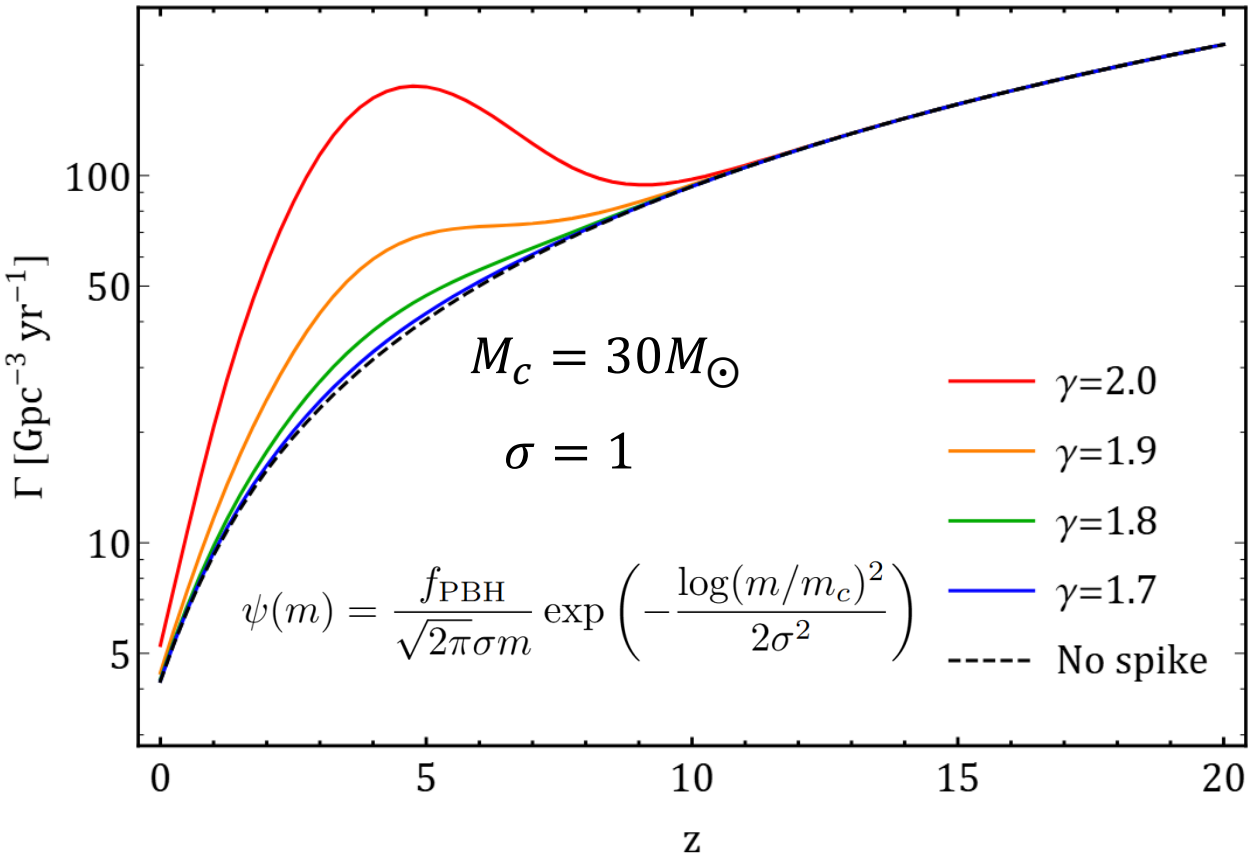




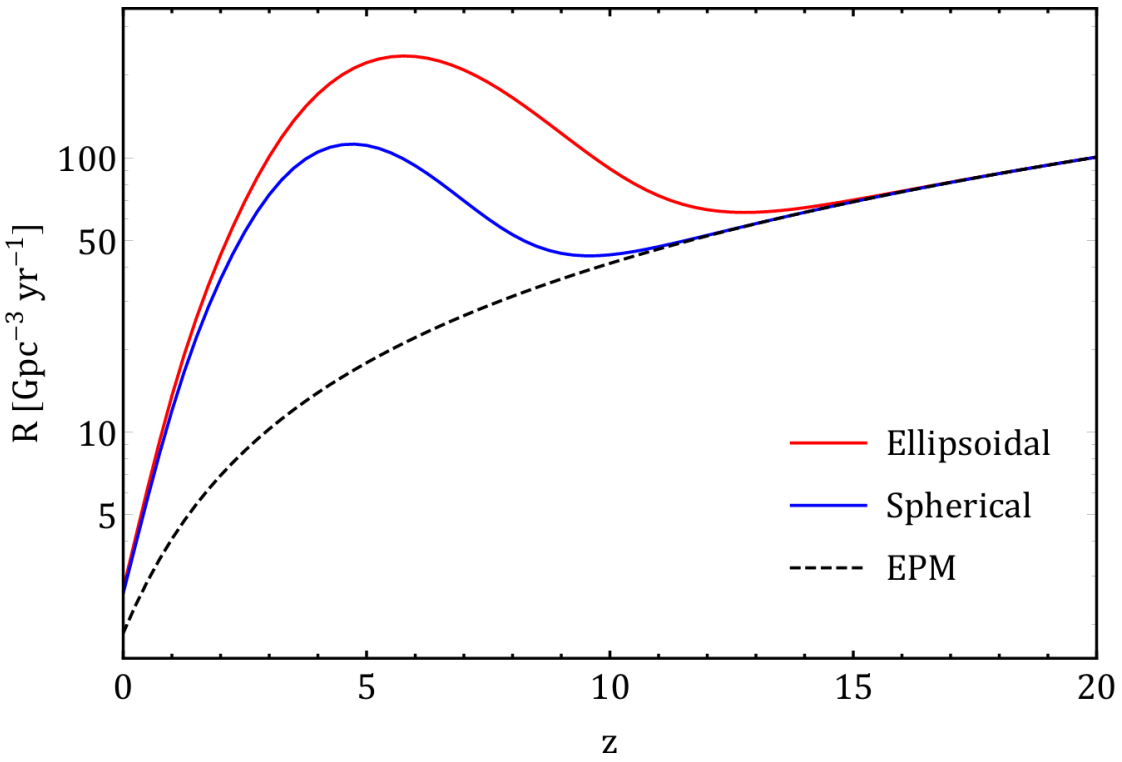
$$R_{\rm tot} = R_{\rm eb} + R_{\rm sp} + R_{\rm halo} + R_{\rm cluster} + \text{etc.}$$
$$\simeq R_{\rm eb} + R_{\rm sp}$$

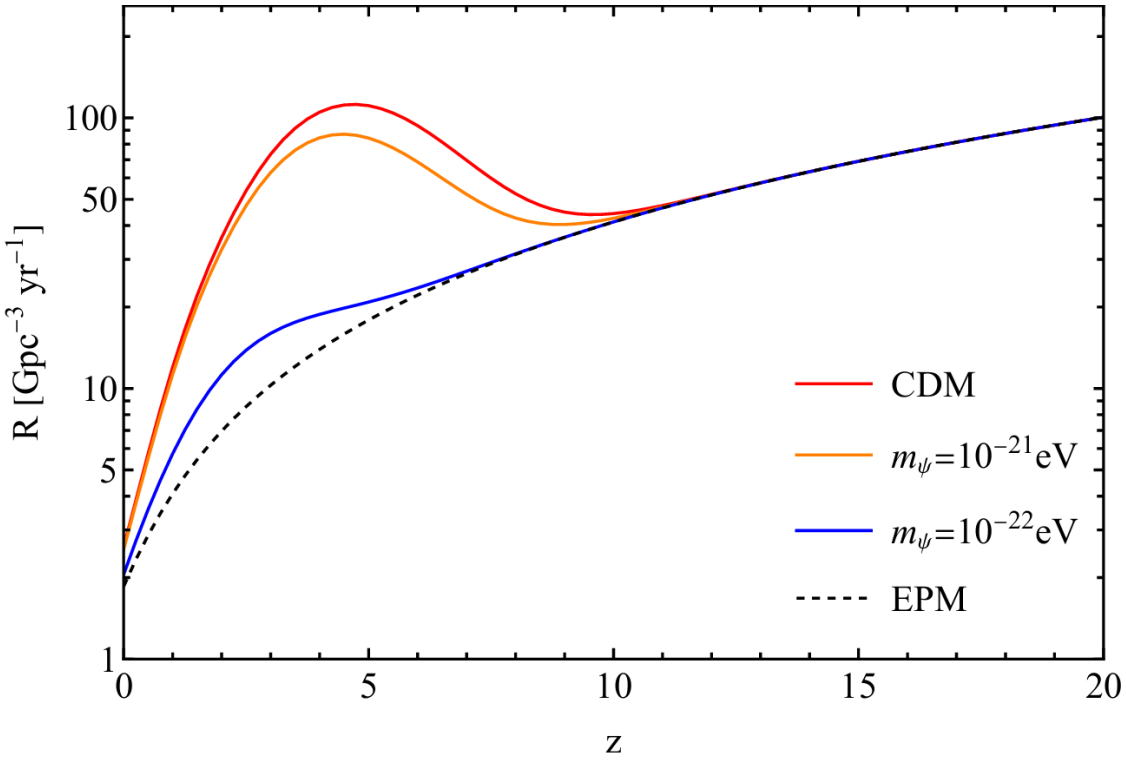
PBH Mass Impact





DM Halo Formation History



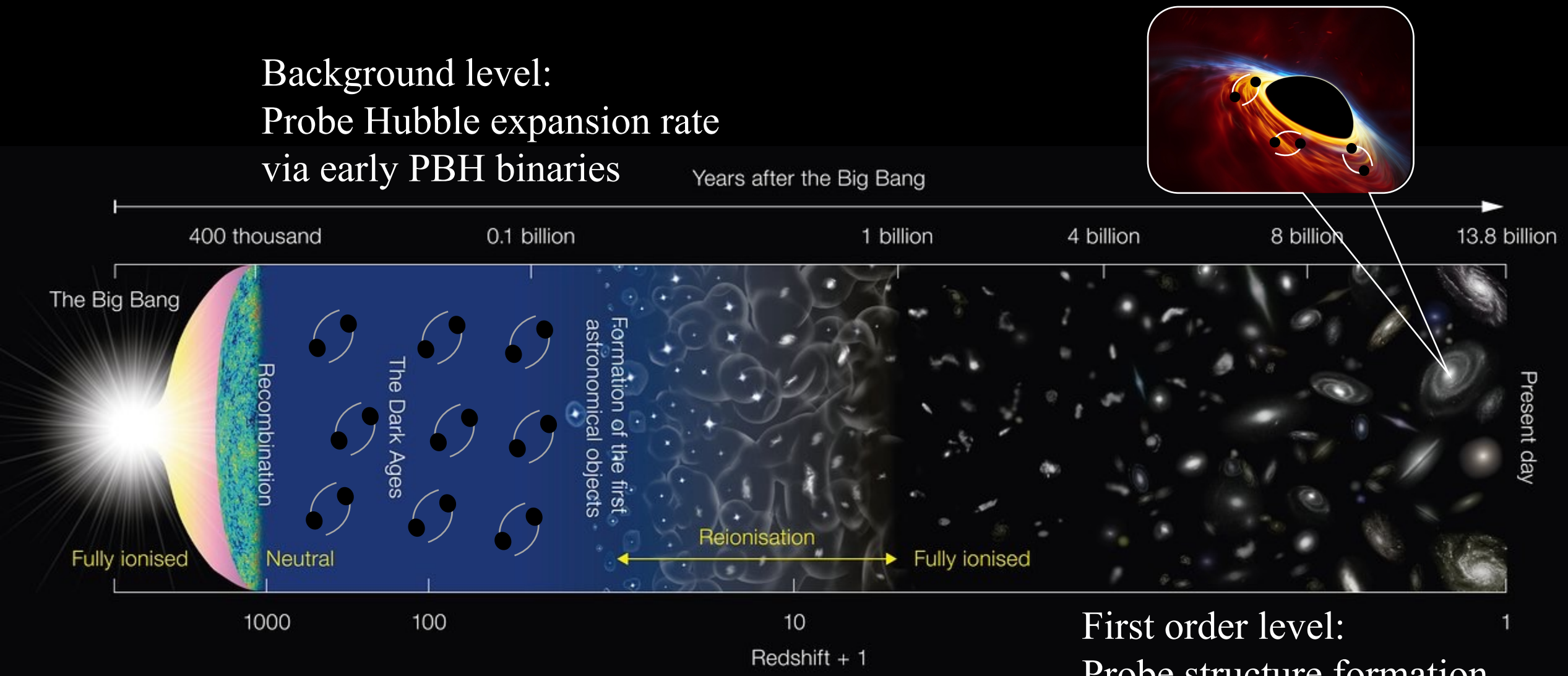


$$f(\sigma_M) = A\left(1+\left(\frac{\sigma_M}{b}\right)^{-a}\right)\exp\left(-\frac{c}{\sigma_M^2}\right)$$

$$f_{\rm ST}(\sigma) = F\sqrt{\frac{2a}{\pi}}\left[1+\left(\frac{\sigma^2}{a\delta_{\rm c}^2}\right)^{p}\right]\frac{\delta_{\rm c}}{\sigma}\exp\left(-\frac{a\delta_{\rm c}^2}{2\sigma^2}\right)$$

$$\left.\frac{dn(M,z)}{dM}\right|_{\psi{\rm DM}} = \left.\frac{dn(M,z)}{dM}\right|_{\rm CDM}\left[1+\left(\frac{M}{M_0}\right)^{-1.1}\right]^{-2.2}$$

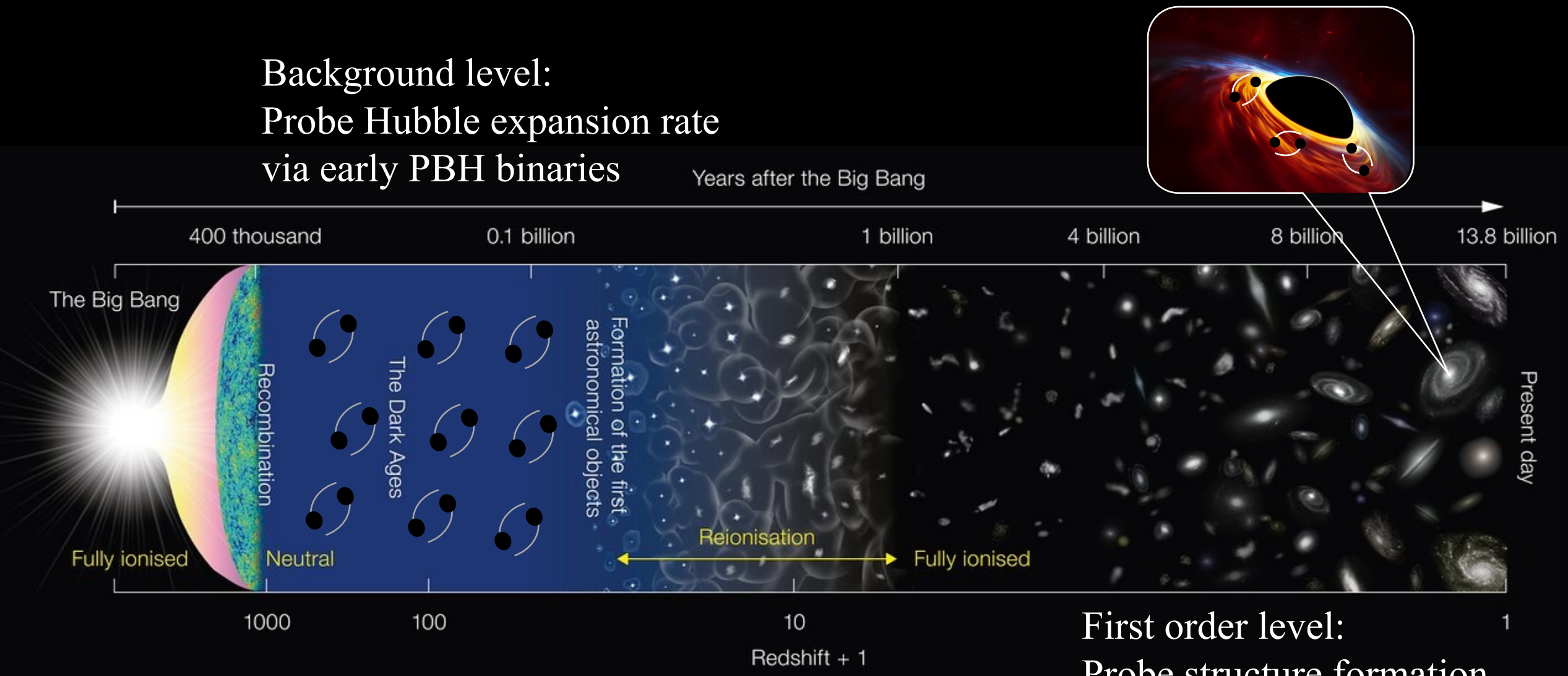
Background level:
Probe Hubble expansion rate
via early PBH binaries



First order level:
Probe structure formation
via late PBH binaries

Image Credit: NAOJ

Background level:
Probe Hubble expansion rate
via early PBH binaries



First order level:
Probe structure formation
via late PBH binaries

Thank You!

Image Credit: NAOJ