# Merger rate of Primordial Black Hole binaries as a probe of Hubble parameter

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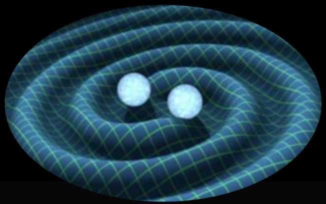
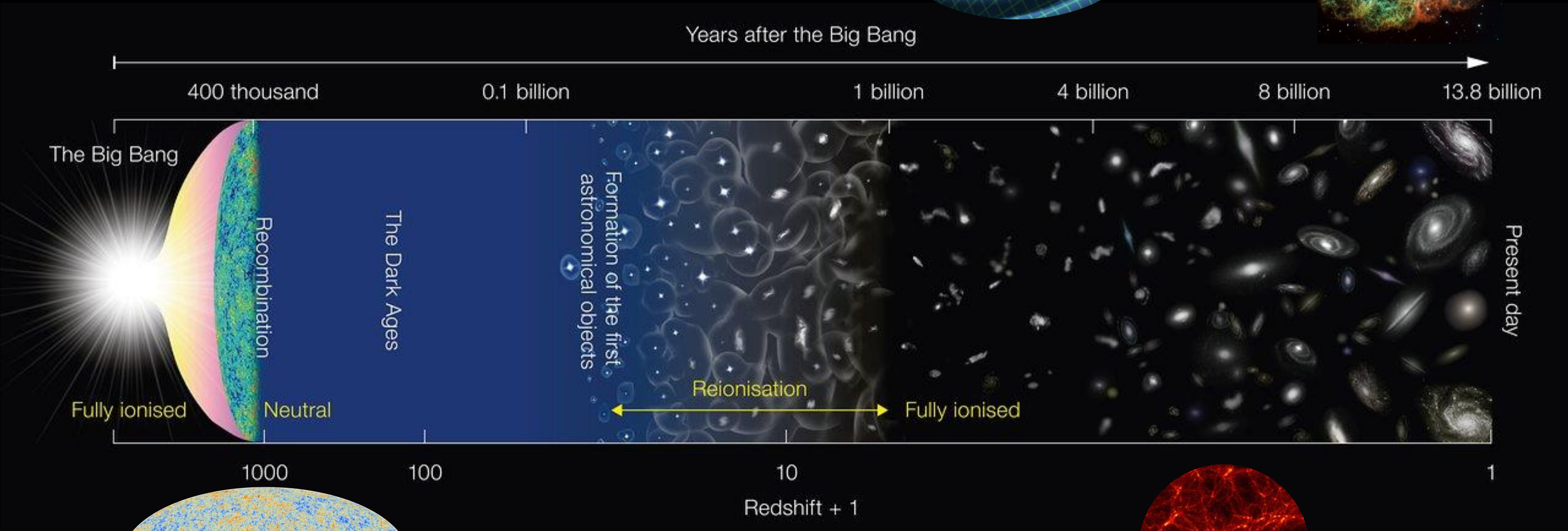
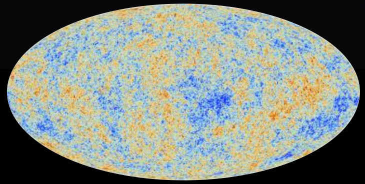
Center for Theoretical Physics of the Universe
Cosmology, Gravity and Astroparticle Physics

$73.04 \pm 1.04$
km/s/Mpc





$67.4 \pm 0.5$
km/s/Mpc

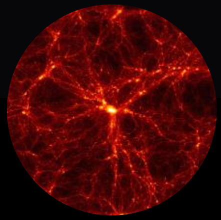
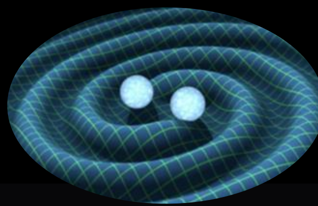
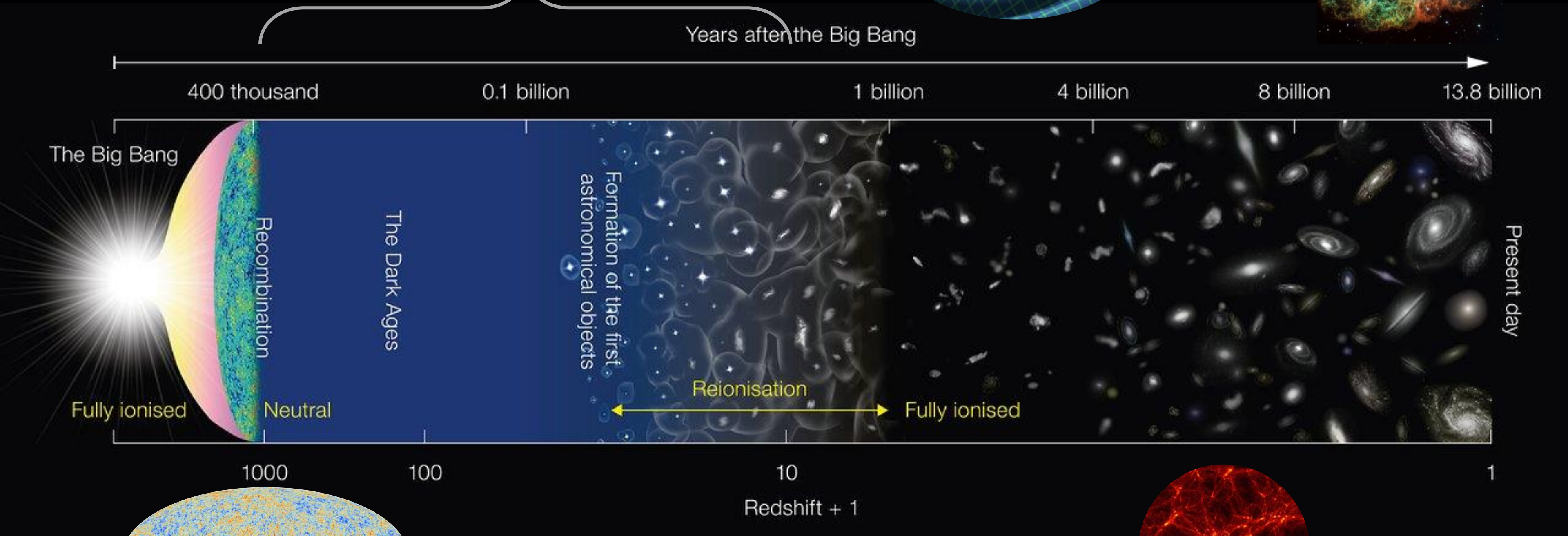
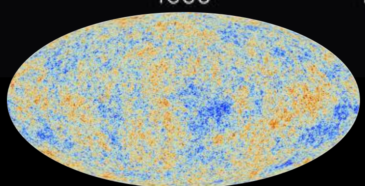


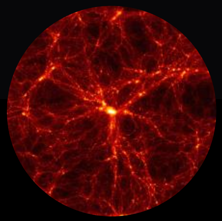
Image Credit: NAOJ

73.04 ± 1.04 km/s/Mpc





67.4 ± 0.5 km/s/Mpc

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# Primordial black holes as a potential candidate

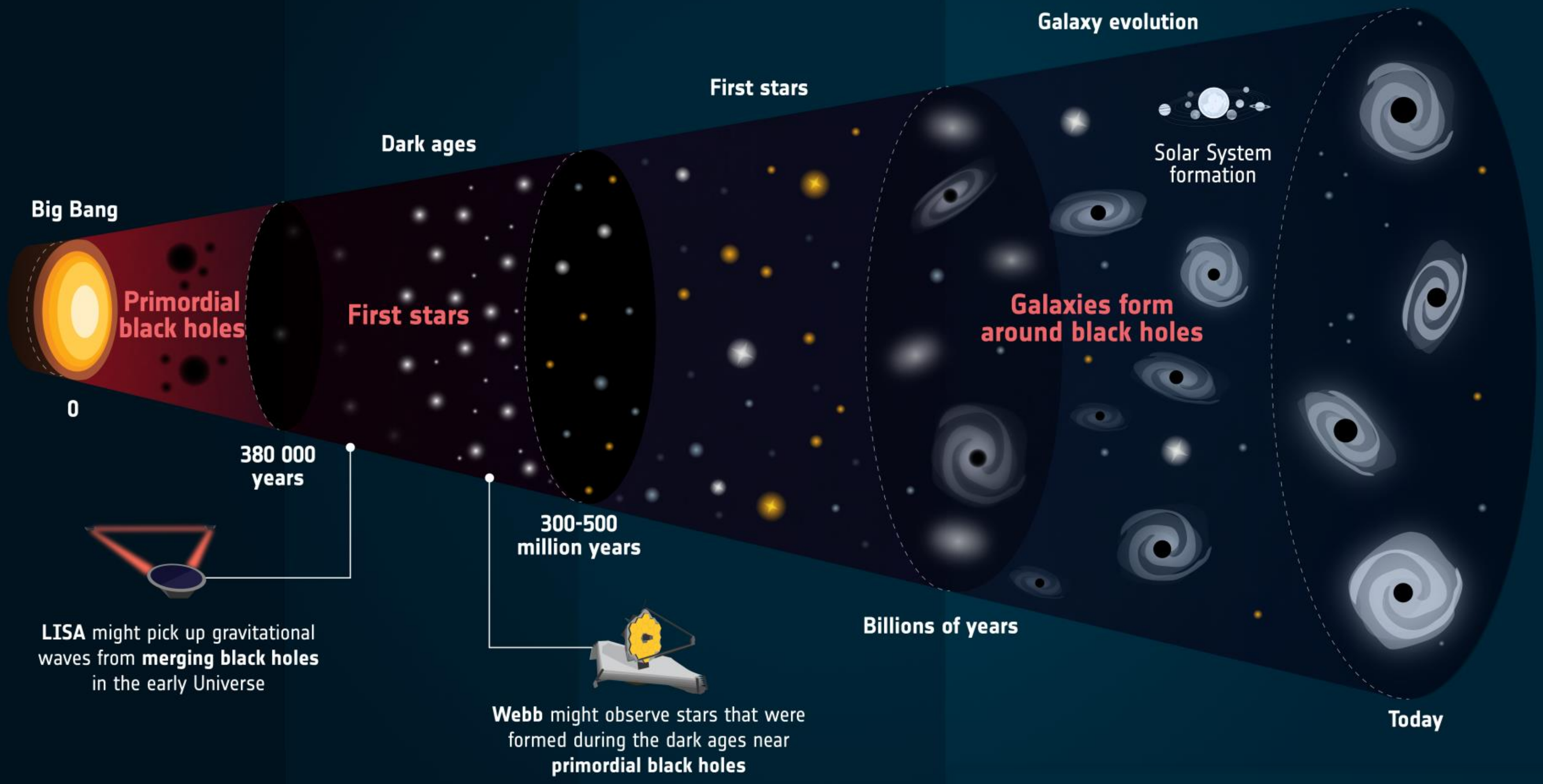


Image Credit: ESA

How to observe the signals from PBHs?

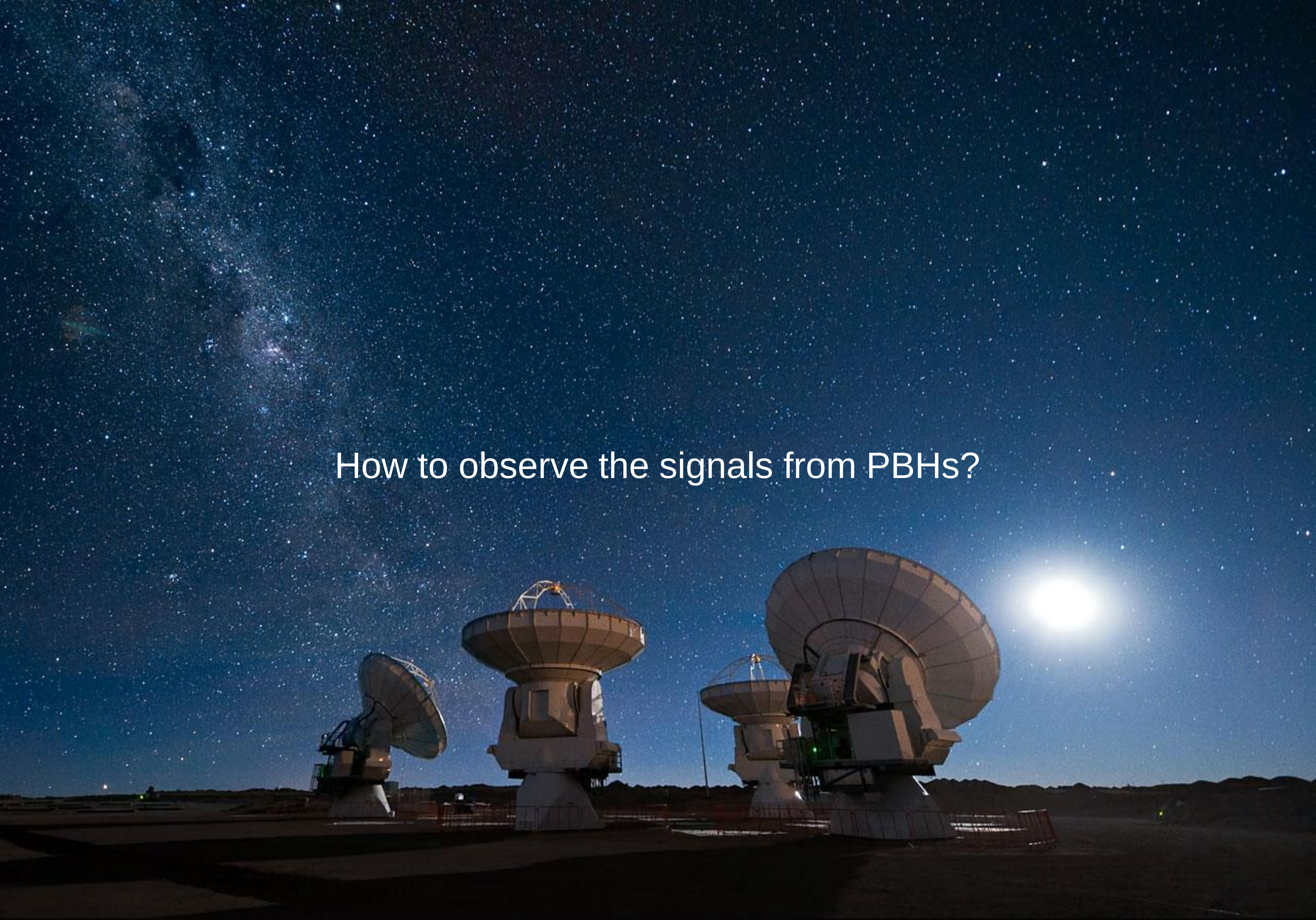

Image Credit: ESO

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$$h(t) = \frac{4}{d_L(z)} \left( \frac{G\mathcal{M}_z}{c^2} \right)^{5/3} \left( \frac{\pi f(t)}{c} \right)^{2/3} \cos \Phi(t)$$

$$d_L = \frac{1+z}{H_0} \int_0^z \frac{c}{E(z')} dz' \qquad \mathcal{M}_z = \mathcal{M}(1+z)$$

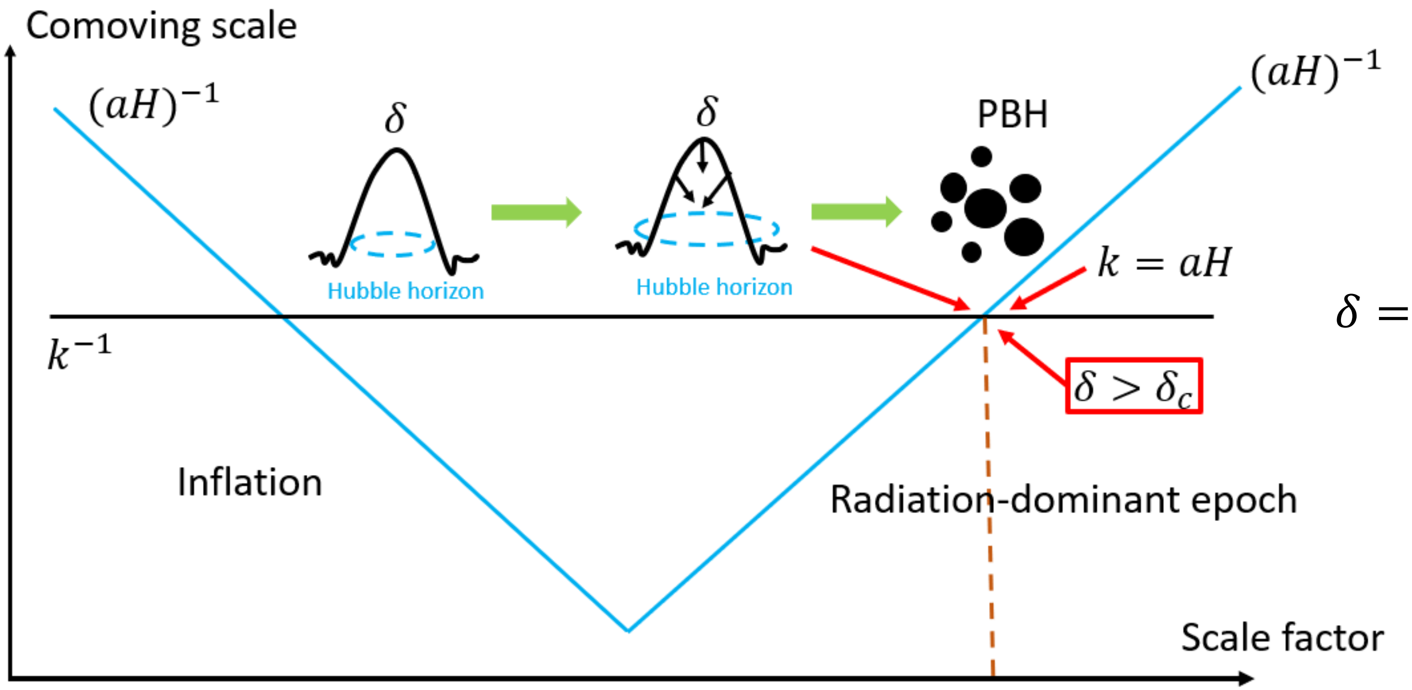
Image Credit: ESO

How to construct redshift-distance relation?

How to construct redshift-distance relation?

A statistical study on PBH binaries may help

# PBH formation




$$\delta = \frac{\delta\rho}{\rho}$$

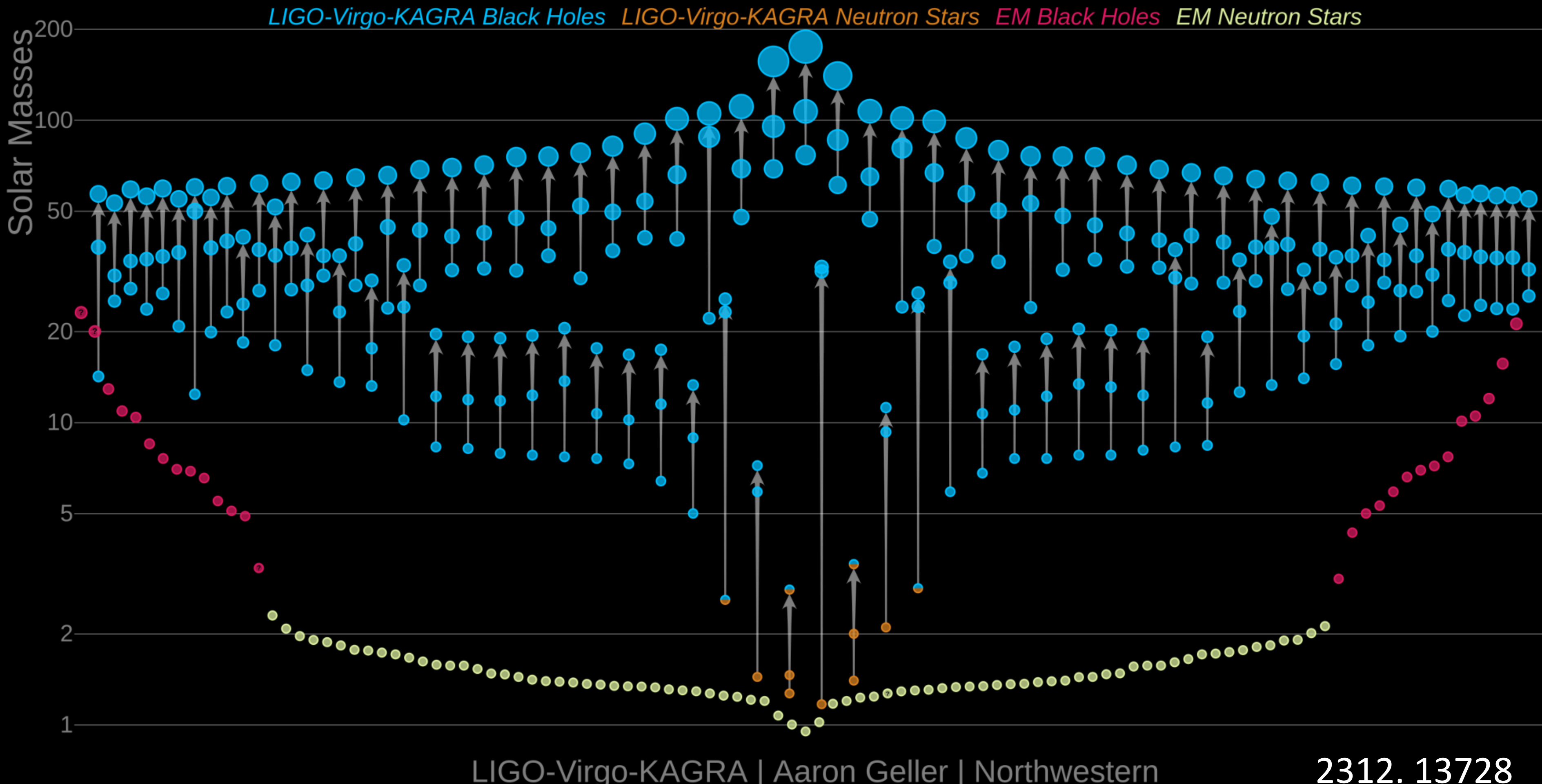
The primordial origin gives an identical primordial mass function

$$n(M) = \frac{f_{\rm PBH}}{\sqrt{2\pi}\sigma M} \exp\left[-\frac{\ln^2(M/M_{pk})}{2\sigma^2}\right]$$

# Merger rate of PBH binaries as a probe of Hubble parameter

## PBH mass function

$$n(M) = \frac{1}{\sqrt{2\pi}\sigma M}\exp\left[-\frac{\ln^2(M/M_{pk})}{2\sigma^2}\right]$$

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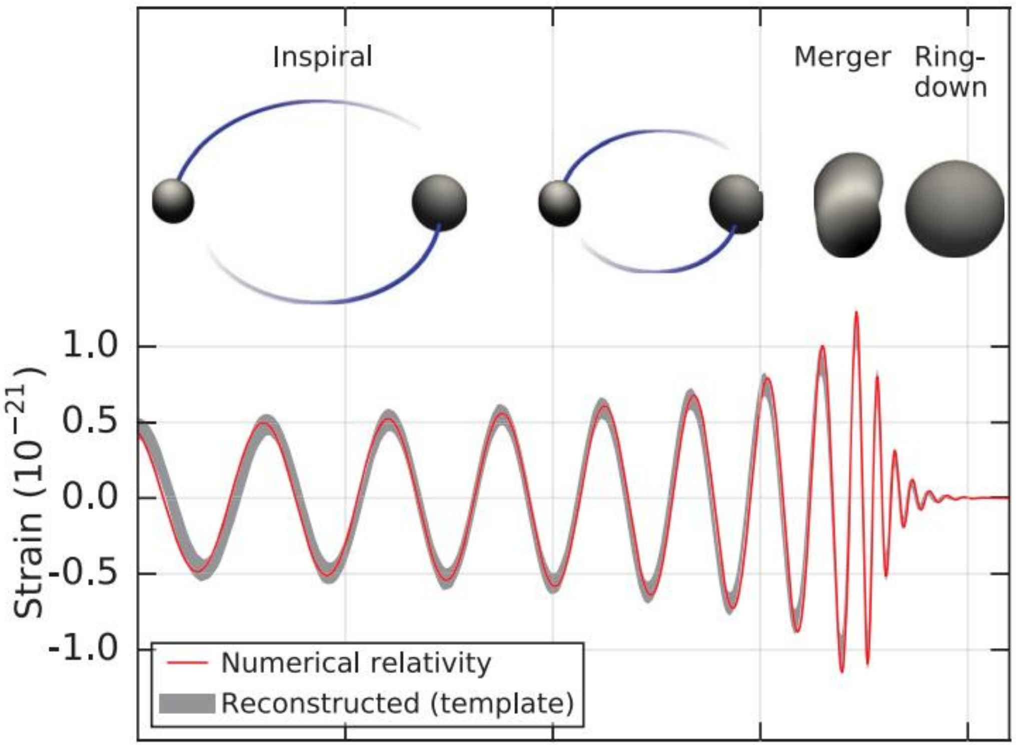
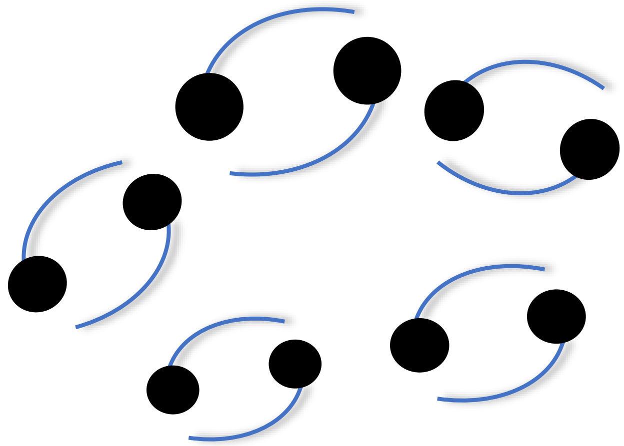



$$\mathcal{M}_z = (1+z)\mathcal{M}$$

$$d_L = \frac{1+z}{H_0}\int_0^z \frac{c}{E(z')}dz'$$

B. P. Abbott et al. Observation of Gravitational Waves from a Binary Black Hole Merger. Phys. Rev. Lett., 116(6):061102, 2016.

$$h(t) = \frac{4}{d_L(z)}\left(\frac{G\mathcal{M}_z}{c^2}\right)^{5/3}\left(\frac{\pi f(t)}{c}\right)^{2/3}\cos\Phi(t)$$



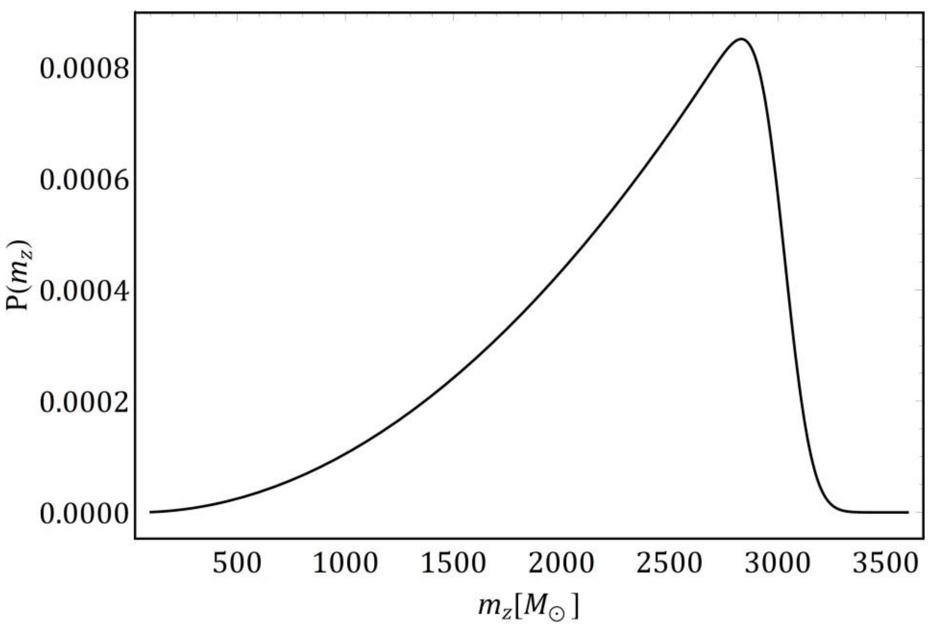
$(m_z^i, d_L^i)$

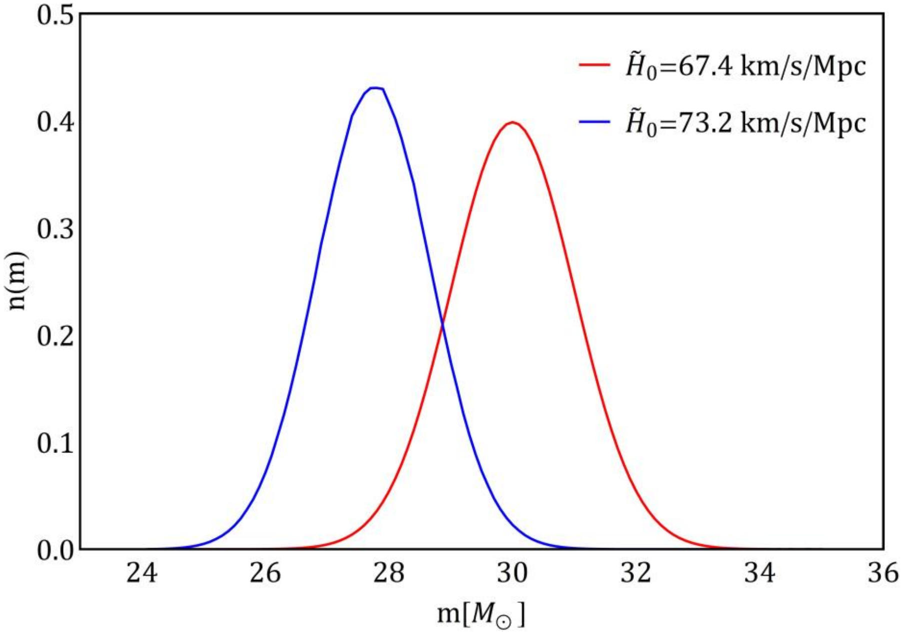
$$d_L^i = \frac{1+z_i}{H_0}\int_0^{z_i}\frac{c}{E(z')}dz'$$

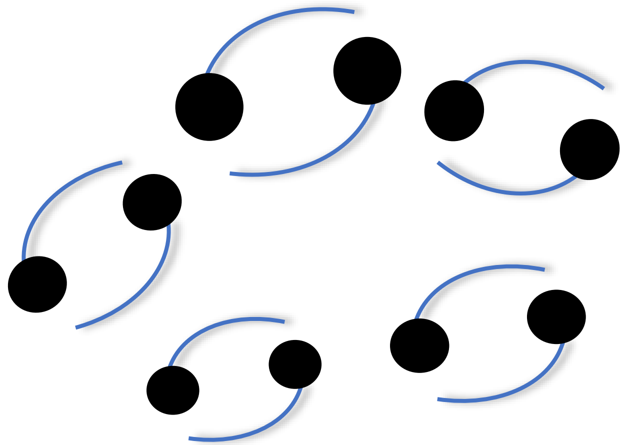
Assume a Hubble parameter $\widetilde{H}_0$

$$z_i = z(d_L^i; \widetilde{H}_0)$$

$$m^i = m_z^i/(1+z_i)$$





$(m_z^i, d_L^i)$

$$d_L^i = \frac{1+z_i}{H_0}\int_0^{z_i} \frac{c}{E(z')}dz'$$

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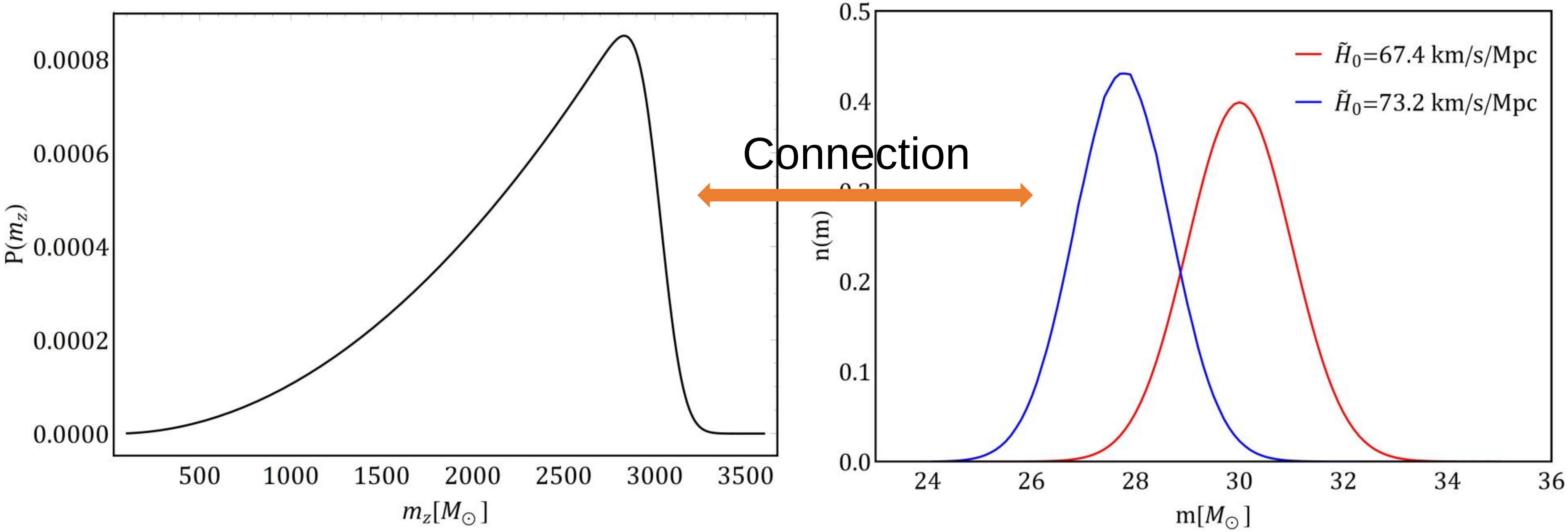


Cumulative distribution

$$C(m_1^z, m_2^z) = \frac{N(m < m_1^z, m_2^z)}{N_{\mathrm{tot}}}$$

detectable window function

$$C(m_1^z, m_2^z) = \int_0^\infty \int_0^{\frac{m_1^z}{1+z}} \int_0^{\frac{m_2^z}{1+z}} n(m_1) n(m_2) W(m_1, m_2; z) p(z)\, dm_1 dm_2 dz$$

PBH mass function

redshift distribution

Probability distribution

$$P(m_1^z, m_2^z) = \frac{1}{N_{\mathrm{tot}}} \frac{dN}{dm_1^z\, dm_2^z}$$

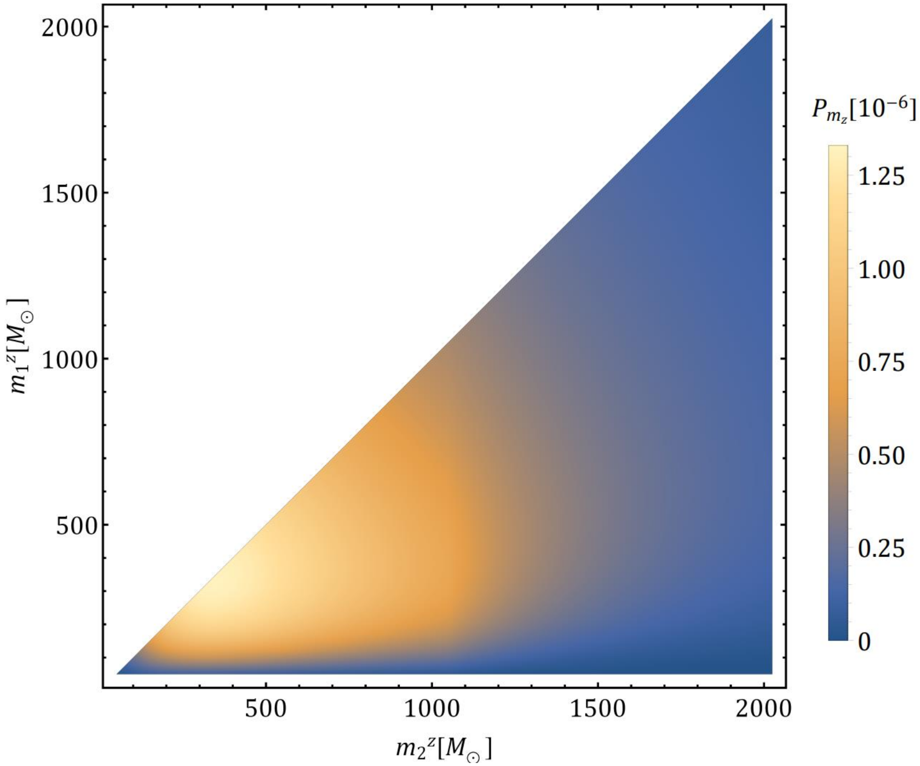
$$P(m_1^z, m_2^z) = \frac{\partial^2 C(m_1^z, m_2^z)}{\partial m_1^z \partial m_2^z}$$

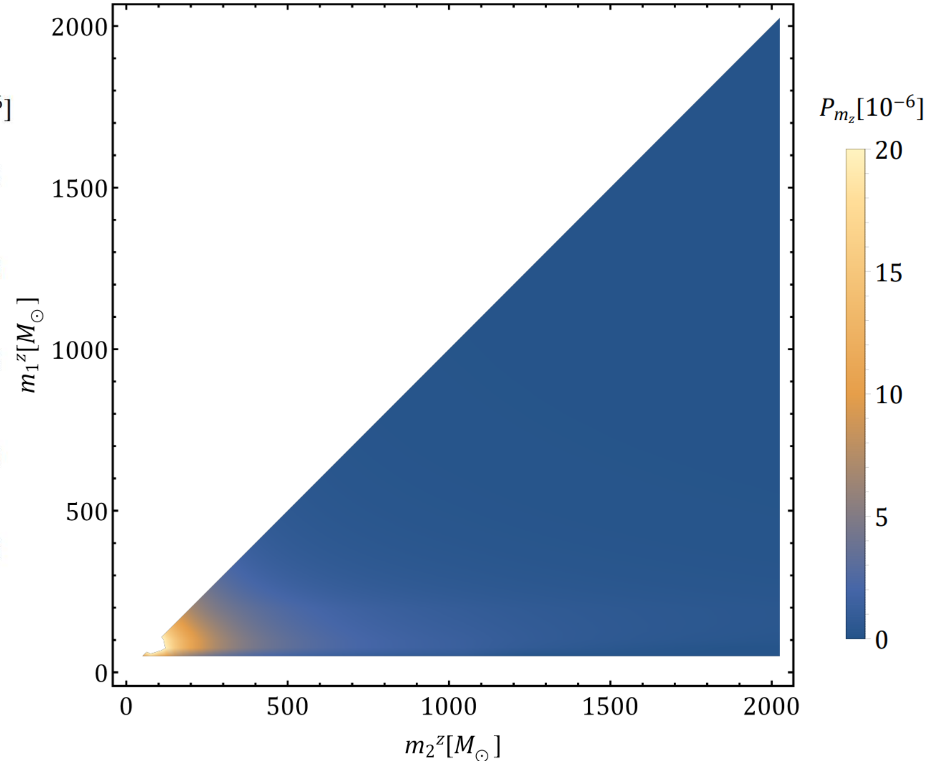
$$P(m_1^z, m_2^z) = \int_0^\infty n\left(\frac{m_1^z}{1+z}\right) n\left(\frac{m_2^z}{1+z}\right) W\left(\frac{m_1^z}{1+z}, \frac{m_2^z}{1+z}; z\right) \frac{p(z)}{(1+z)^2}\, dz$$

$$W(m_1, m_2; z) = \frac{N_{\mathrm{obs}}(m_1, m_2; z)}{N_{\mathrm{tot}}(m_1, m_2; z)} = \int_{a_{\min}}^{a_{\max}} \int_{e_{\min}}^{e_{\max}} P(a, e; z)\, dade$$

$$\mathrm{SNR} = \sqrt{4 \int_{f_{\min}}^{f_{\max}} \frac{\left|\tilde{h}(f)\right|^2}{S_n(f)} df} > 8 \qquad \tilde{h}(f) = \sqrt{\frac{5}{24}} \frac{(G\mathcal{M}_z)^{5/6}}{\pi^{2/3} c^{3/2} d_L} f^{-7/6}$$

$$p(z) \propto \frac{\dot{n}(z)}{1+z} \frac{dV_c}{dz} \qquad \dot{n}(z) \propto \left(\frac{t(z)}{t_0}\right)^{-34/37}$$




$$n(m) = \frac{1}{\sqrt{2\pi}\sigma m}\exp\left[-\frac{\ln^2(m/m_{pk})}{2\sigma^2}\right]$$

$$n(m) = \frac{\alpha-1}{M}\left(\frac{m}{M}\right)^{-\alpha}$$

$$P_O(m_1^z, m_2^z) = \int_0^\infty n_p\left(\frac{m_1^z}{1+z}\right) n_p\left(\frac{m_2^z}{1+z}\right) W\left(\frac{m_1^z}{1+z}, \frac{m_2^z}{1+z}; z\right) \frac{p(z)}{(1+z)^2}\, dz$$

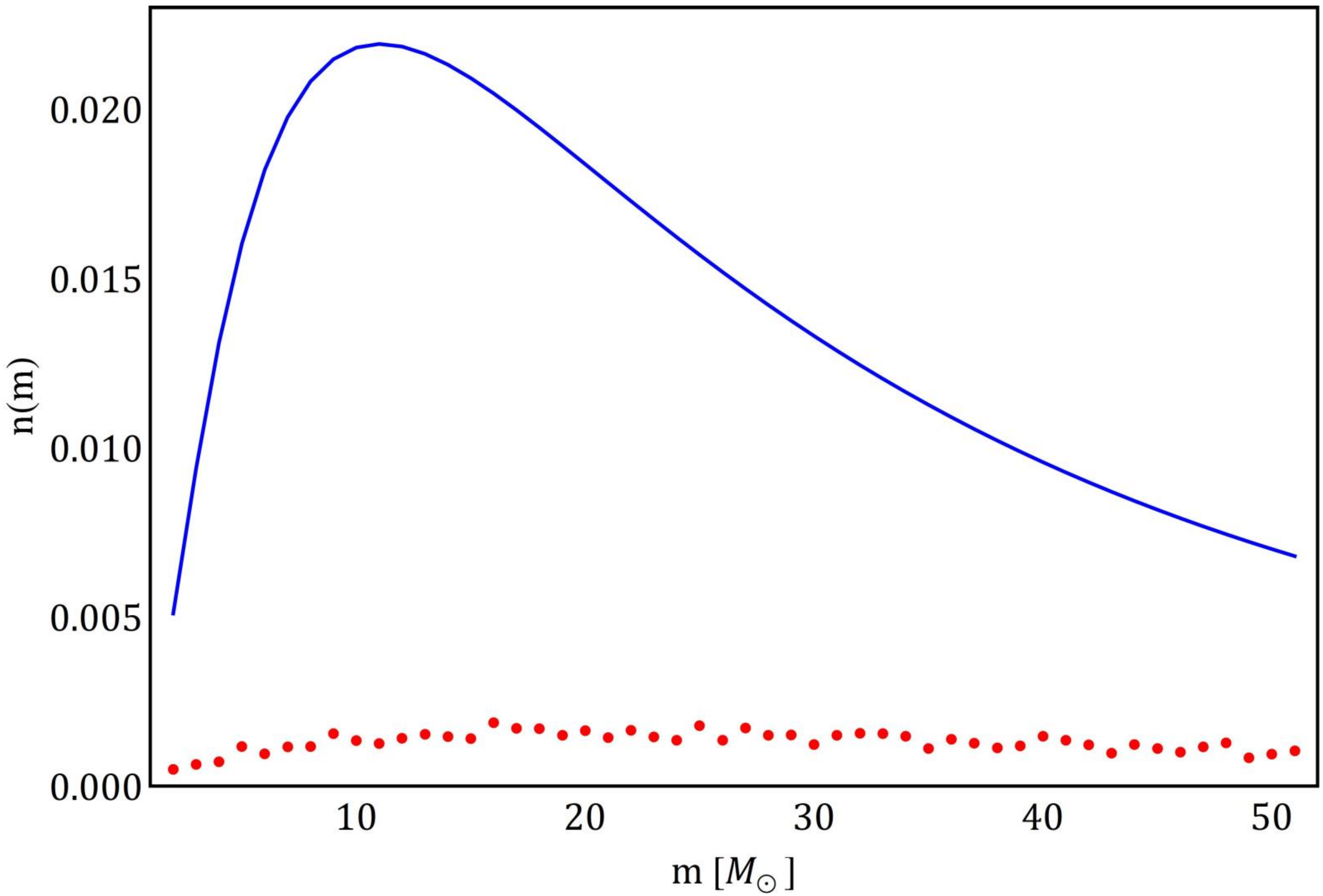
# Gradient Descent Method

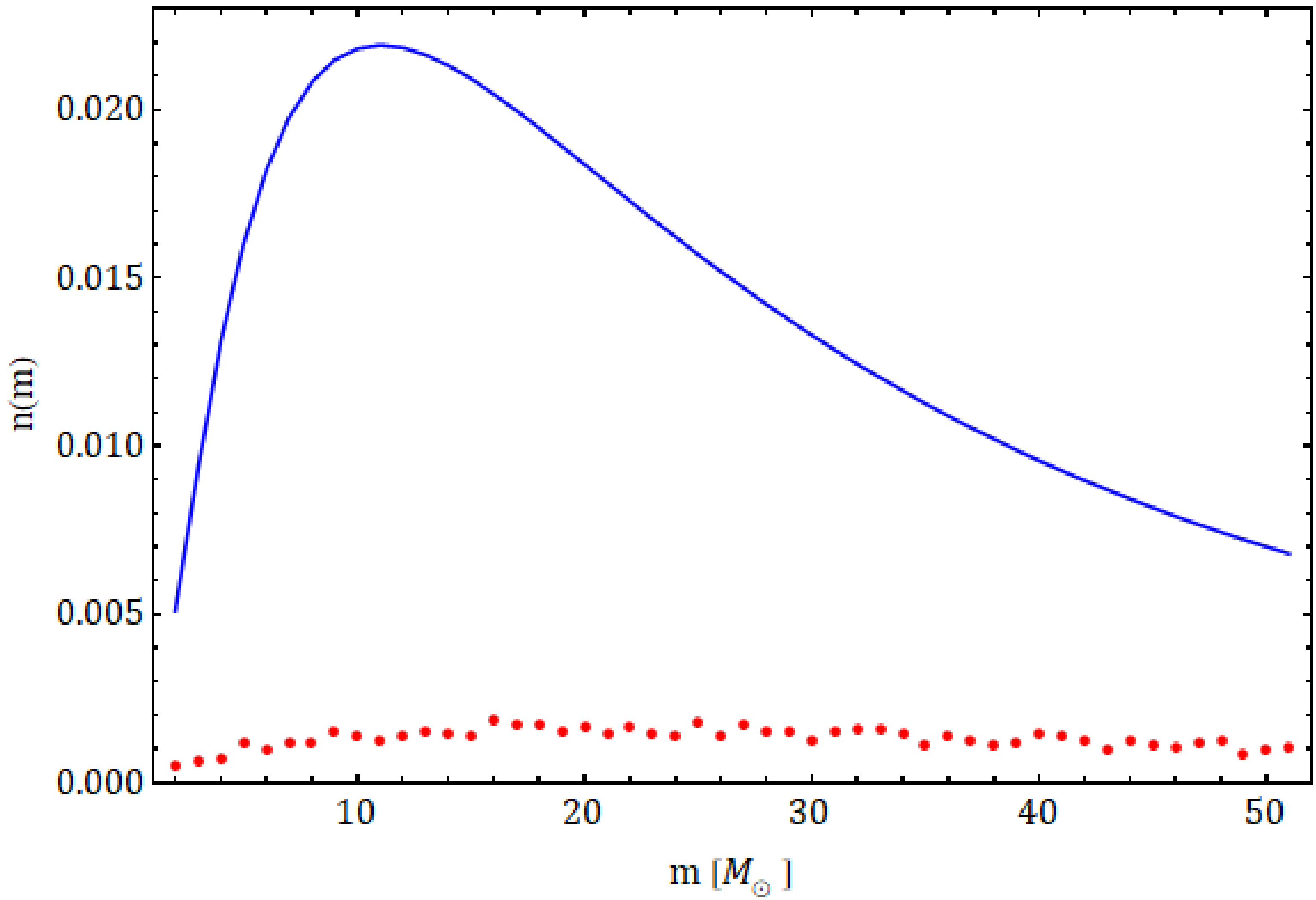
$$P_O(m_1^z, m_2^z) = \int_0^\infty n_p\left(\frac{m_1^z}{1+z}\right) n_p\left(\frac{m_2^z}{1+z}\right) W\left(\frac{m_1^z}{1+z}, \frac{m_2^z}{1+z}; z\right) \frac{p(z)}{(1+z)^2}\, dz$$

$$P_T(m_1^z, m_2^z) = \int_0^\infty n'\left(\frac{m_1^z}{1+z}\right) n'\left(\frac{m_2^z}{1+z}\right) W\left(\frac{m_1^z}{1+z}, \frac{m_2^z}{1+z}; z\right) \frac{p(z)}{(1+z)^2}\, dz$$

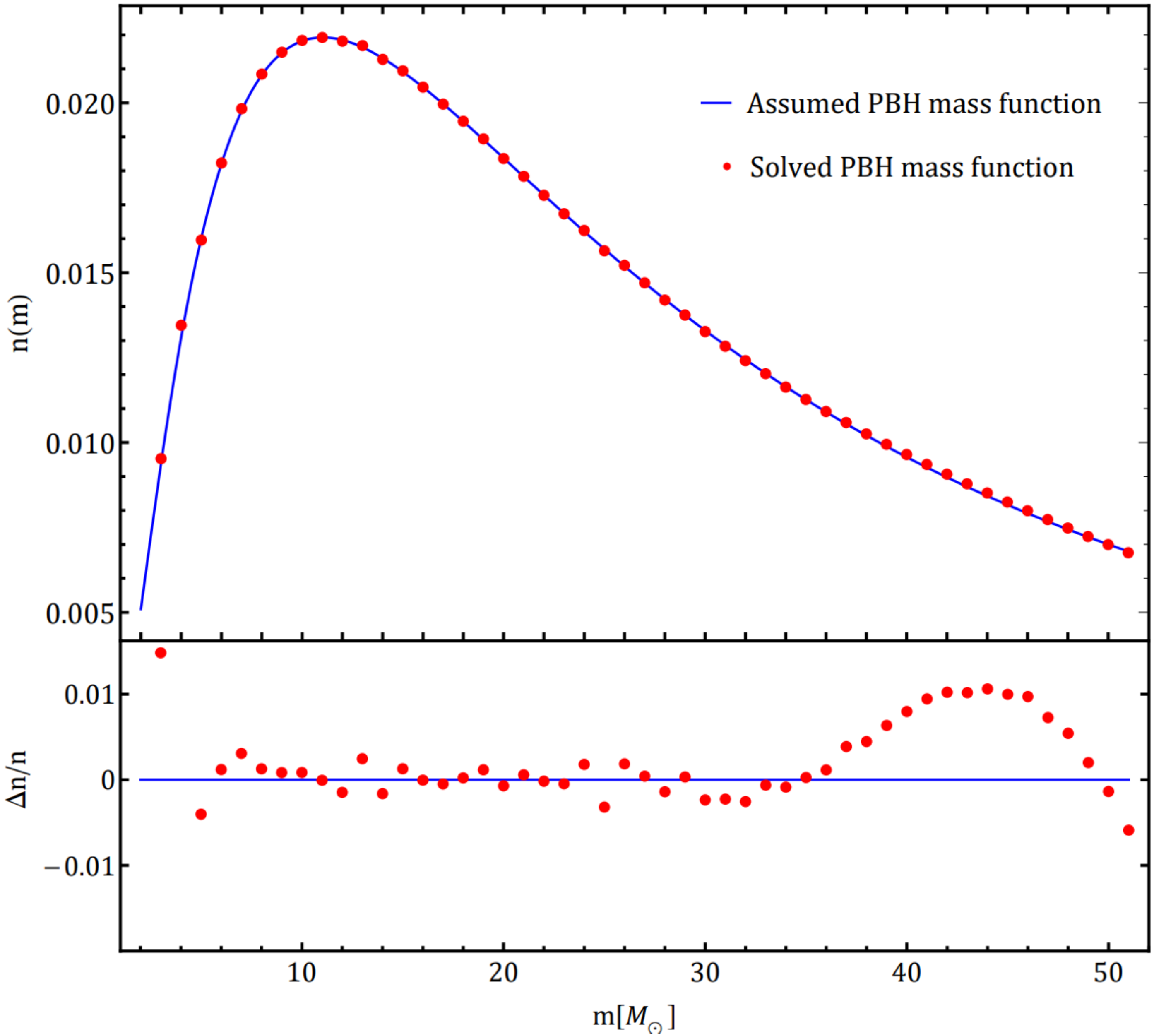
$$E(n) = \sqrt{\frac{\sum_{1 \le i \le j, j=1}^{j=N} [P_T(m_1^z, m_2^z) - P_O(m_1^z, m_2^z)]^2}{(1+N)N/2}}$$

$$n_{k+1}(m_i) = n_k(m_i) - \gamma \frac{\partial E(n_k)}{\partial n_k(m_i)}$$


0.020
0.015
0.010
0.005
0.000
n(m)
10
20
30
40
50
m [$M_\odot$]



n(m)
0.020
0.015
0.010
0.005
0.000
10
20
30
40
50
m [$M_\odot$]



Assumed PBH mass function
Solved PBH mass function
n(m)
0.020
0.015
0.010
0.005
Δn/n
0.01
0
−0.01
10
20
30
40
50
m[$M_\odot$]

How about $p(z)$?

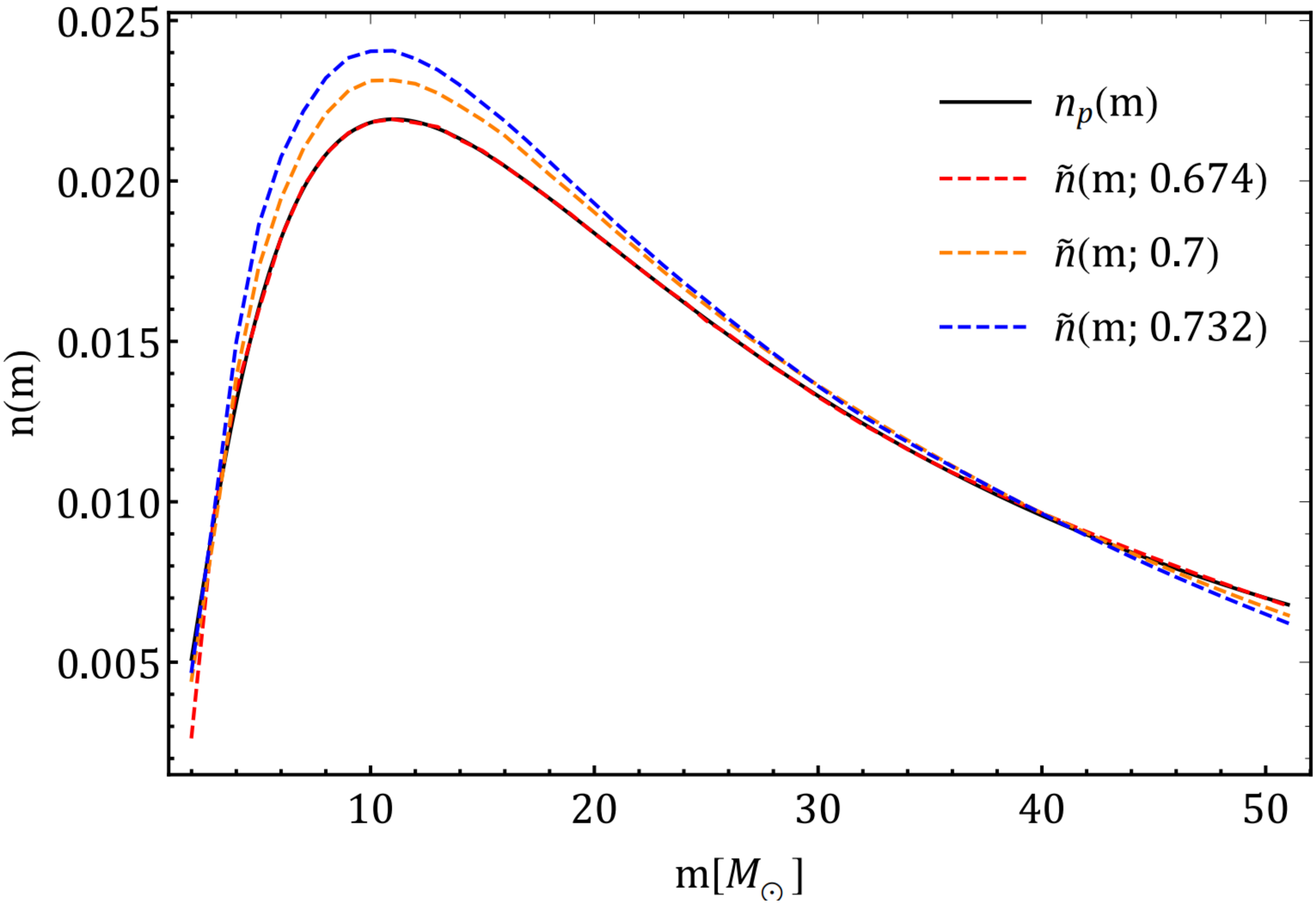
$$d_L^i = \frac{1+z_i}{H_0}\int_0^{z_i}\frac{c}{E(z')}dz'$$

Assume a Hubble parameter $\widetilde{H}_0$

$$z_i = z(d_L^i;\widetilde{H}_0)$$

$$\left.\right\} \quad p(z;\widetilde{H}_0)$$

$$P_O(m_1^z, m_2^z) = \int_0^\infty \tilde{n}\left(\frac{m_1^z}{1+z}\right)\tilde{n}\left(\frac{m_2^z}{1+z}\right)W\left(\frac{m_1^z}{1+z},\frac{m_2^z}{1+z};z\right)\frac{p(z;\widetilde{H}_0)}{(1+z)^2}dz$$



0.025
0.020
0.015
0.010
0.005
n(m)
10
20
30
40
50
m[$M_\odot$]
$n_p$(m)
$\tilde{n}$(m; 0.674)
$\tilde{n}$(m; 0.7)
$\tilde{n}$(m; 0.732)

However, we don’t know the PBH mass function currently.

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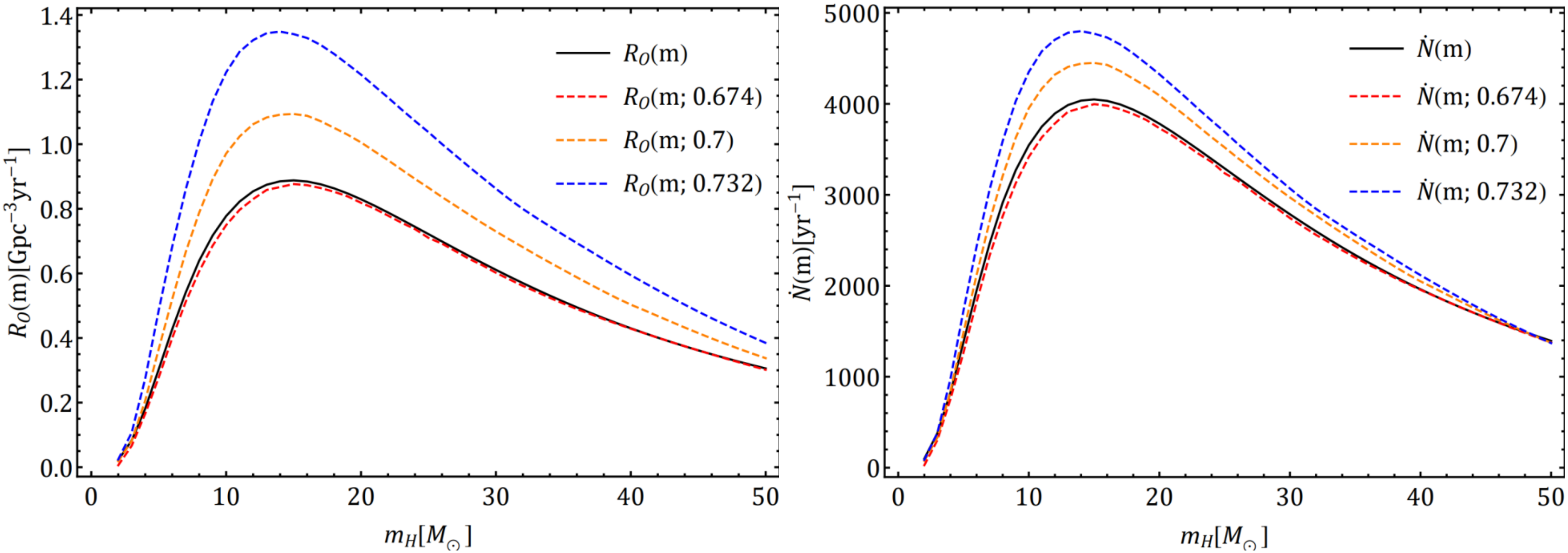
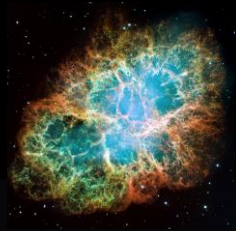
Another observable related with PBH mass function
Merger rate of PBH binaries

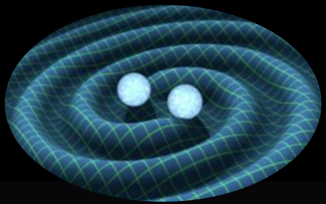



$$R_{ij} = \rho_{\mathrm{PBH}} \min\left(\frac{n(m_i)}{m_i}, \frac{n(m_j)}{m_j}\right) \Delta_m \frac{dP}{dt}$$

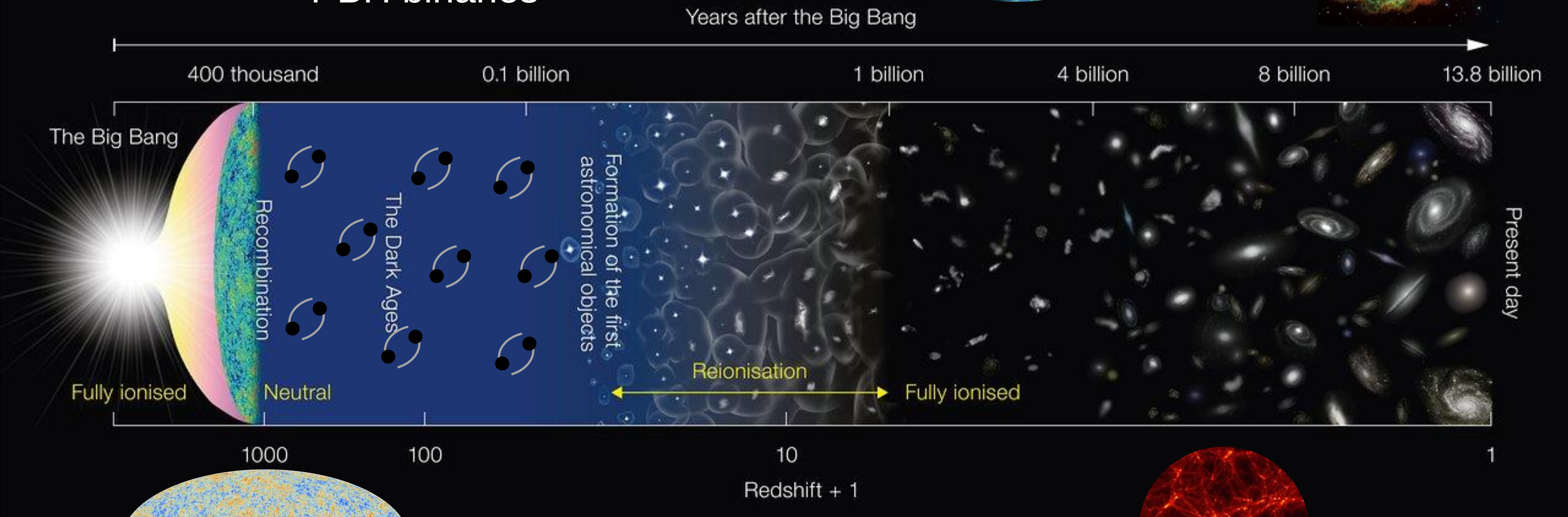
$$\dot{N} = \int_{z_{\min}}^{z_{\max}} \frac{R}{1+z} \frac{dV_c}{dz} dz$$

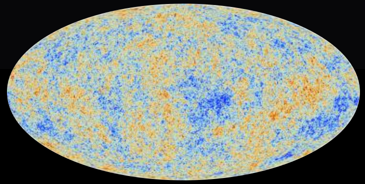
73.04 ± 1.04 km/s/Mpc

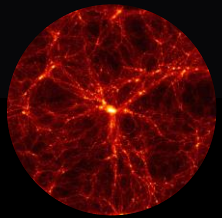




PBH binaries



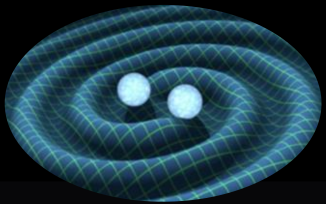


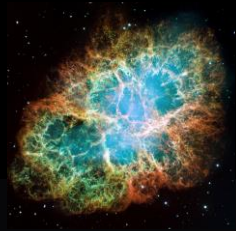
67.4 ± 0.5 km/s/Mpc

Image Credit: NAOJ

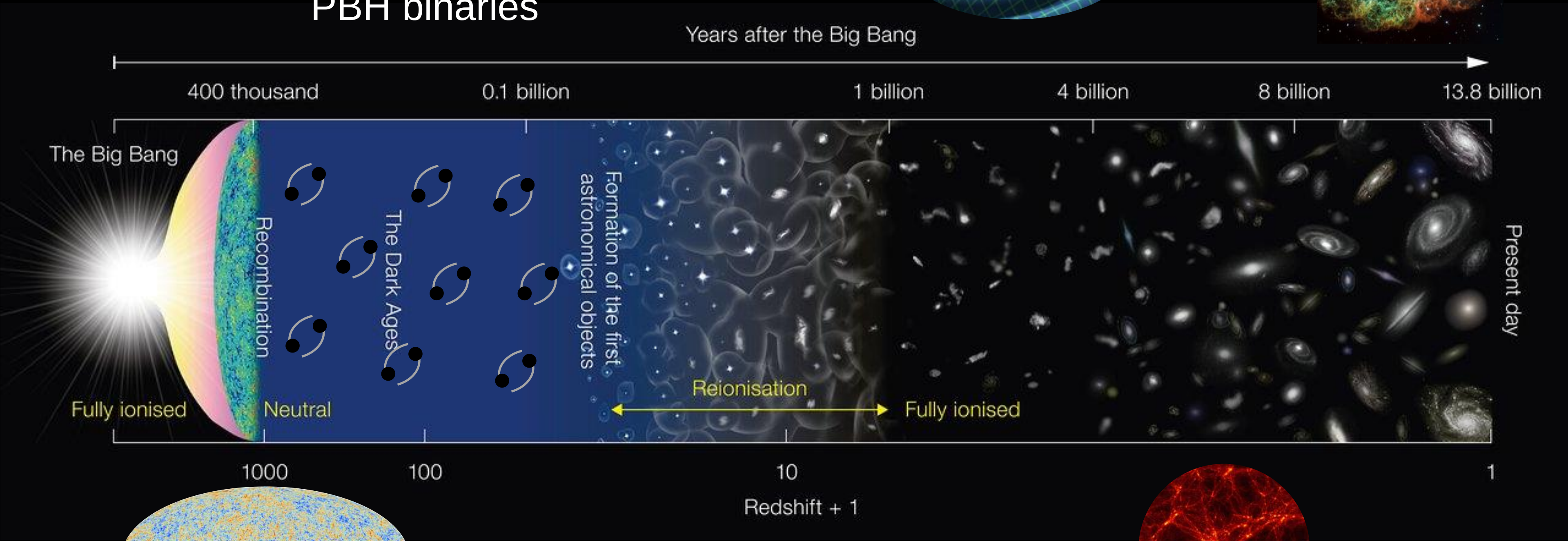
# Thank you!

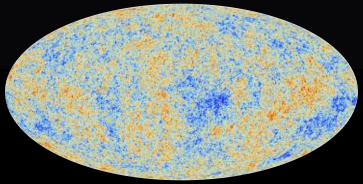
$73.04 \pm 1.04$ km/s/Mpc

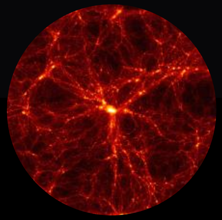




PBH binaries





$67.4 \pm 0.5$ km/s/Mpc

Image Credit: NAOJ

$$P_O(m_1^z, m_2^z) = \int_0^\infty n_z\left(\frac{m_1^z}{1+z}\right) n_z\left(\frac{m_2^z}{1+z}\right) W\left(\frac{m_1^z}{1+z}, \frac{m_2^z}{1+z}; z\right) \frac{p(z)}{(1+z)^2}\, dz$$

$$n_z(m_z) = n_{\mathrm{i}}(m_i)\frac{dm_{\mathrm{i}}}{dm_z} = n_{\mathrm{i}}(m_i)\, g(z, m_{\mathrm{z}})$$

$$\frac{dm}{dt} = 4\pi\lambda\rho_m \frac{G^2 m^2}{v_{\mathrm{eff}}^3}$$

$$P_O(m_1^z, m_2^z) = \int_0^\infty n_{\mathrm{i}}\left(\frac{m_1^z}{1+z}\right) n_{\mathrm{i}}\left(\frac{m_2^z}{1+z}\right) g(z, \frac{m_1^z}{1+z}) g(z, \frac{m_2^z}{1+z})$$
$$\times W\left(\frac{m_1^z}{1+z}, \frac{m_2^z}{1+z}; z\right) \frac{p(z)}{(1+z)^2}\, dz$$